A Dynamic Cost Analysis in an Inventory Model with Partially Time Varying Demand

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Abstract:
In this inventory model, we study the inventory model with time dependent demand without deterioration of items in present model. The fluctuation in inventory level is affected by only time and demand. Shortages are also not considered in this system. Effects of different parameter on holding cost, ordering cost are analyzed in this model. The main objective to study this model is to evaluate the total cost, to study the significant effect on profit. Model validation is given by numerical consideration which show the nature of the cost with respect to different level of variations in parameters.

Keywords: Inventory, demand, holding cost, ordering cost

Introduction

The inventory management is very crucial parts of business to streamline development of particular inventory. Effective management can lead for industry to develop a clear vision, tracking leadership, and freehand communication, leading the businesses towards progress in neck cutting environment in modern time. By managing resource allocation, satisfaction of customer, and individual development, management improving efficiency and innovation. A sound financial practices ensure good resilience and a continuous improvement gives lifelong success. With principles of optimization a businesses can neglect uncertainties, profiting the opportunities, and achieve a standing development while in any dynamic and fighting environments. The inventory level impose a significantly role in business's economy, level of industry in market capturing. Storage of goods is difficult managing up to notable demand and improving inventory sale to make optimize profit in challenging environment. Product utility and visibility is main factors of demand in market. Showcasing of items can create advantage in business. Product showcasing and availability of items play lead roles in increasing public interest. Everyone knows that the industrial product always have a finite lifetime and some deterioration. Items like medicine and electronic gadgets have long durability period but fruits and vegetables have very small life spans. Manager must have careful attention to prevent perishable inventory ensure optimal use. Effective inventory management can establish a balance policy for supply and demand chain system, dynamic environments, anticipating market trends and reducing wastages.

Many of researcher have been developed different inventory model to enhance industry policies. Researcher Ghare. and Schrader [9] have developed an inventory model for exponentially decaying inventory system. Covert and Philip [7] has given an EOQ model with weibull distribution deterioration. Haley and Higgin [12] have been published an inventory policy under consideration of trade credit financing environment system. In the early stages of inventory development, researcher Philip [19] has given a generalized EOQ inventory model for items having deterioration as weibull pattern. Also, Donaldson [8] gives inventory replenishment policy for a linear trend in demand. He has also authenticate the model by numerical analysis. Aggarwal [1] has published a note on an order level inventory model for a system with constant rate of deterioration. Bhunia and Maiti [2] have analyzed many deterministic inventory models under consideration of variable production rates. Chakrabarti and Chaudhuri [3] have studied an EOQ model for deteriorating items with a linear trend in demand with shortages in inventory for all cycles. Shah and Jaiswal [20] have developed an order level inventory model for a system having constant rate of deterioration. Jamal, Sarkar and Wang [15] have given an ordering policy for deteriorating items with allowable shortage and permissible delay in payments. Chang and Dye [4] have developed an EOQ model for deteriorating items with time varying demand and partial backlogging. Ghasemi and Damghani [10] have given an article on topic on robust simulation-optimization approach for pre-disaster multi-period location–allocation–inventory planning. Goyal and Giri [11] have published a research article for recent trends in modelling of deterioration inventory. Sharma and Kumar [21] have been presented an optimum ordering interval policy with known demand for items with variable rate of deterioration and shorages are also allowed in the model. Sharma and Kumar [22] have given also an optimum ordering interval policy with known demand for items with variables rate of deterioration and shortages. Ouyang, Wu and Cheng [18] have published an article for inventory model with deteriorating items with exponentially declining demand and partial backlogging is also included in system. Teng, Cheng and Ouyang [23] have been given in inventory model for deteriorating items with power form stock dependent demand. Chun- Tao Chang, Goyal, Jinn – Tsair Teng [5] have developed an inventory model for economic order quantity policy for perishable items under stock dependent selling rate and time dependent partial backlogging. Mehta, Niketa and Shah [17] have been published order level lot size inventory model for deteriorating items with exponentially decreasing demand. Chung-Yuan Dye, Tsu- Pang Hsieh and Liang- Yuh Ouyng [6] have given an article on determining optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging. Jaggi and Mittal [13] published an EOQ model for deteriorating items with time dependent demand under inflationary conditions. Jaggi, Kausar and Khanna [14] has published two stage credit policy as joint optimization of Retailers unit selling price and fixed cycle length, when the end demand is price as well as credit period sensitive. Mark, Vaidy, Gilvan [16] have given a note on an application of the EOQ model with nonlinear holding cost to inventory management of perishables items.

Here, we developed an EOQ model having no deterioration of inventory and demand varying as time. In this inventory model we have calculated the total expenditures on different field in a fixed cycle time and studied the variations due to all parameters on total cost. We have validated this model by numerical analysis.

Assumption and Notation

1. C_1 = Setup cost per unit item.
2. C_2 = Holding cost per unit item per unit time.
3. D(t) = demand = $\begin{cases} td, & 0 < t < t_1 \\ d, & t_1 \le t \le T \end{cases}$.
4. d = demand parameter for items.
5. T = Fixed cycle time.
6. Q = Initially inventory Level.
7. I(t) = Inventory level at time t.
8. $C(t_1, T)$ = Total cost per cycle time.
9. TD = Total demand in cycle time.
10. O = Ordering cost per order per unit.
11. H = Holding cost in cycle time.
12. TC = Total cost in cycle time.

Mathematical Model

We are examining a perpetual inventory system that operates indefinitely and involves demand that varies up to a time limit after that it will constant till end of cycle time. Our policy does not allow the inventory deterioration. Let I(t) represent the level of inventory at any given time t. The producing of inventory initiated at time t = 0. The procurement, management and sale with other activities commence at the beginning of production. During a cycle time, if manufacturing ceases, the inventory level rapidly declines towards zero at time T. The mathematical policy for a system is as follows:

$$\frac{d}{dt} I(t) = td, \qquad 0 \le t \le t_1 \qquad \text{...(1)}$$

$$\frac{d}{dt} I(t) = d + I(t), \; t_1 \le t \le T \qquad \text{...(2)}$$

The boundary conditions are $I(0) = 0,\ I(T) = Q$.

Solution of differential equation (1) is $I(t)=\frac{t^2}{2}d$

Solution of differential equation (2) is $I(t)=(dt+c_2)(1+t+t^2)$

After using the boundary condition $I(t)=-dt+Q\left(1-T+\frac{T^2}{2}\right)$

From the above two solutions, we find $Q=\frac{d}{2}t_1(3+2t_1)(2+2T+T^2)$

The total demand in cycle time [0, T] is given as D= $\int_0^T D(t)dt \quad =\frac{d}{6}(3T^2-t_1^2)$

Number of items in duration [0, T]

$$=\int_0^T I(t)dt = \frac{d}{6}\left\{(t_1^2-T^2)(t_1^2-2t_1+T+t_1T)+Tt_1(4-5T+t_1)\right\}$$

Now we calculate the different cost related to this inventory model

Ordering cost = QC = $C_1\frac{d}{2}t_1(3+2t_1)(2+2T+T^2)$

Holding cost = HC = $C_2\int_0^T I(t)dt$

$$=\frac{C_2 d}{6}\left\{(t_1^2-T^2)(t_1^2-2t_1+T+t_1T)+Tt_1(4-5T+t_1)\right\}$$

The total cost per unit time per unit item is given as

$C(t_1,T)=$ $\frac{1}{T}$ (Ordering cost + Holding cost)

$$=C_1\frac{d}{2T}t_1(3+2t_1)(2+2T+T^2)+\frac{C_2 d}{6T}\left\{(t_1^2-T^2)(t_1^2-2t_1+T+t_1T)+Tt_1(4-5T+t_1)\right\}$$

Now to find optimal solution, we apply the condition of optimality on cost that $\frac{\partial C(t_1,T)}{\partial t_1}=0=\frac{\partial C(t_1,T)}{\partial T}$. And determine stationary values of t_1 and T as t_1^*,T^*. At these stationary points, we find $\frac{\partial^2 C(t_1,T)}{\partial t_1^2},\frac{\partial^2 C(t_1,T)}{\partial T^2},\frac{\partial^2 C(t_1,T)}{\partial t_1\partial T}$

.

If $\frac{\partial^2 C(t_1,T)}{\partial t_1^2}\times\frac{\partial^2 C(t_1,T)}{\partial T^2}>\left(\frac{\partial^2 C(t_1,T)}{\partial t_1\partial T}\right)^2$ then the cost will be minimum.

Numerical Example with Graph

We study the effect of different parameters on total cost,

Table 1

d	t_1	T	cost
50	1.9	5	106.09
52	1.9	5	115.34
54	1.9	5	126.57
56	1.9	5	140.31
58	1.9	5	152.21
60	1.9	5	167.94
62	1.9	5	184.85
64	1.9	5	199.34
66	1.9	5	216.28

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68	1.9	5	238.51

Tabel 2

d	t_1	T	cost
50	1.9	5	106.09
50	1.87	5	107.32
50	1.86	5	110.29
50	1.84	5	113.26
50	1.83	5	114.71
50	1.82	5	113.24
50	1.81	5	115.21
50	1.79	5	116.38
50	1.77	5	117.57
50	1.74	5	118.36

Table 3

d	t_1	T	cost
50	1.9	5	106.09
50	1.9	5.2	113.29
50	1.9	5.4	119.64
50	1.9	5.6	127.37
50	1.9	5.8	138.61
50	1.9	6	151.24
50	1.9	6.2	165.28
50	1.9	6.4	184.74
50	1.9	6.6	201.28
50	1.9	6.8	227.84

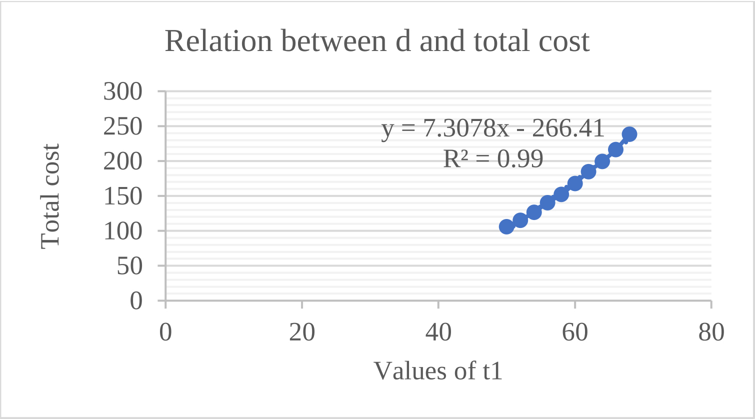

Figure 1

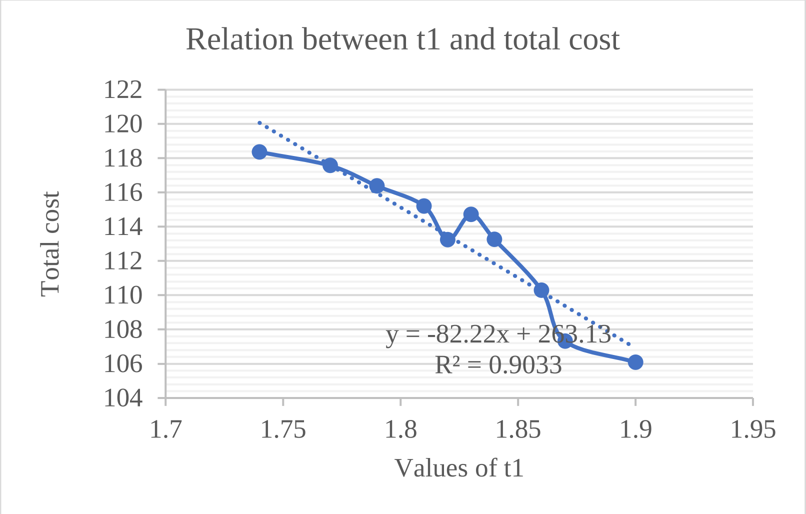

Figure 2

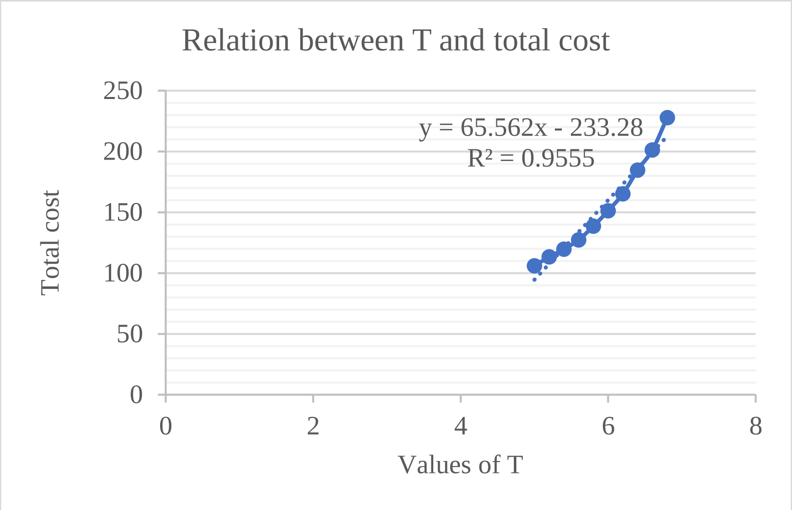

Figure 3

Conclusions and Remarks:

The manufacturing rate of goods to replenish the stock in this research study is proportionate to the fluctuating demand over limited time after it become constant. It is very clear that production rate directly depends on demand satisfaction in the market. Demand implies that client pleasure is essential. It is evident from the preceding numerical computation and graph analysis that the overall cost rises as d and T rises but decreases as t_1 grow; in the numerical example, these values are at the lowest level. These reduce the overall expense. We might improve our inventory system and make significant profits based on the considered facts. In practical situations, production, cost, sales, and profitability are all significantly impacted by these elements. Our operations' productivity depends on several variables, including government taxes, labor costs, workforce size, and equipment infrastructure. Our inventory system may be impacted by additional factors. These will be included in our upcoming research.

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