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Bipolar Vague α Generalized Connectedness in Topological Spaces

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Abstract: In this paper, we have introduced the basic concepts of bipolar vague α generalized connected space, bipolar vague α generalized super connected space and bipolar vague α generalized extremally disconnected space and discussed some of their properties.

Keywords: Bipolar vague topology, bipolar vague α generalized closed set, bipolar vague α generalized continuous mapping, bipolar vague α generalized connected space, bipolar vague α generalized super connected space and bipolar vague α generalized extremally disconnected space.

1. Introduction

Fuzzy set was introduced by L.A.Zadeh [15] in 1965. The concept of fuzzy topology was introduced by C.L.Chang [3] in 1968. The generalized closed sets in general topology were first introduced by N.Levine [11] in 1970. K.Atanassov [2] in 1986 introduced the concept of intuitionistic fuzzy sets. The notion of vague set theory was introduced by W.L.Gau and D.J.Buehrer [8] in 1993. D.Coker [6] in 1997 introduced intuitionistic fuzzy topological spaces. On Connectedness in intuitionistic fuzzy special topological spaces was introduced by Ozcag and Coker [12] in 1998. Bipolar- valued fuzzy sets, which was introduced by K.M.Lee [10] in 2000 is an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [-1,1]. A new class of generalized bipolar vague sets was introduced by S.Cicily Flora and I.Arockiarani [4] in 2016. We have introduced bipolar vague α generalized closed sets [13] in 2024. In this paper, we have introduced bipolar vague α generalized connected space, bipolar vague α generalized super connected space and bipolar vague α generalized extremally disconnected space and discussed some of their properties.

2. Preliminaries

Here in this paper the bipolar vague topological spaces are denoted by (X, BV_τ). Also, the bipolar vague interior, bipolar vague closure of a bipolar vague set A are denoted by BVInt(A) and BVCl(A). The complement of a bipolar vague set A is denoted by A^c and the empty set and whole sets are denoted by $0_\sim$ and $1_\sim$ respectively.

Definition 2.1: [10] Let X be the universe. Then a bipolar valued fuzzy sets, A on X is defined by positive membership function μ_A^+, that is μ_A^+: X→ [0,1], and a negative membership function μ_A^-, that is μ_A^-: X→ [-1,0]. For the sake of simplicity, we shall use the symbol A = $\{\langle x, \mu_A^+(x), \mu_A^-(\text{x})\rangle : x \in X\}$.

Definition 2.2: [10] Let A and B be two bipolar valued fuzzy sets then their union, intersection and complement are defined as follows:

(i) $\mu_{A\cup B}^+$= max $\{\mu_A^+(x), \mu_B^+ x)\}$

(ii) $\mu_{A\cup B}^-$= min $\{\mu_A^-(x), \mu_B^- x)\}$

(iii)$\mu_{A\cap B}^+$= min $\{\mu_A^+(x), \mu_B^+ x)\}$

(iv)$\mu_{A\cap B}^-$= max $\{\mu_A^-(x), \mu_B^- x)\}$

(v) $\mu_{A^c}^+$(x) = 1-$\mu_A^+(x)$ and $\mu_{A^c}^-$(x) = -1-$\mu_A^-(x)$ for all $x \in X$.

Definition 2.3: [8] A vague set A in the universe of discourse U is a pair of (t_A, f_A) where t_A: U→[0,1], f_A: U→[0,1] are the mapping such that $t_A + f_A \leq 1$ for all $u \in U$. The function t_A and f_A are called true membership function and false membership function respectively. The interval [t_A, $1 - f_A$] is called the vague value of u in A, and denoted by $v_A(u)$, that is $v_A(u) = [t_A(u), 1 - f(u)]$.

Definition 2.4: [8] Let A be a non-empty set and the vague set A and B in the form A = $\{\langle x, t_A(x), 1 - f_A(x)\rangle : \text{x} \in \text{X}\}$, B = $\{\langle x, t_B(x), 1 - f_B(x)\rangle : \text{x} \in \text{X}\}$. Then

(i) A ⊆ B if and only if $t_A(x) \leq t_B(x)$ and $1 - f_A(x) \leq 1 - f_B(x)$

(ii) A ∪ B = $\left\{\langle \max(t_A(x), t_B(x)), \frac{\max(1 - f_A(x), 1 - f_B(x))\rangle}{\text{x}} \in \text{X}\right\}$.

(iii)A ∩ B = $\left\{\langle \min(t_A(x), t_B(x)), \frac{\min(1 - f_A(x), 1 - f_B(x))\rangle}{\text{x}} \in \text{X}\right\}$.

(iv)A^c = $\{\langle x, f_A(x), 1 - t_A(x)\rangle : \text{x} \in \text{X}\}$.

Definition 2.5: [1] Let X be the universe of discourse. A bipolar-valued vague set A in X is an object having the form A = $\{\langle \text{x}, [t_A^+(x), 1 - f_A^+(x)], [-1 - f_A^-(x), t_A^-(x)]\rangle : \text{x} \in \text{X}\}$ where $[t_A^+, 1 - f_A^+]$: X→[0,1] and $[-1 - f_A^-, t_A^-]$: X→[-1,0] are the mapping such that $t_A^+(x) + f_A^+(x) \leq 1$ and -1≤ $t_A^- + f_A^-$. The positive membership degree $[t_A^+(x), 1 - f_A^+(x)]$ denotes the satisfaction region of an element x to the property corresponding to a bipolar-valued set A and the negative membership degree $[-1 - f_A^-(x), t_A^-(x)]$ denotes the satisfaction region of x to some implicit counter property of A. For a sake of simplicity, we shall use the notion of bipolar vague set $v_A^+ = [t_A^+, 1 - f_A^+]$ and $v_A^- = [-1 - f_A^-, t_A^-]$.

Definition 2.6: [5] A bipolar vague set A = $[v_A^+, v_A^-]$ of a set U with v_A^+= 0 implies that $t_A^+ = 0$, $1 - f_A^+$= 0 and v_A^-= 0 implies that $t_A^- = 0$, $-1 - f_A^- = 0$ for all x ∈ U is called zero bipolar vague set and it is denoted by 0.

Definition 2.7: [5] A bipolar vague set A = $[\nu_A^+, \nu_A^-]$ of a set U with ν_A^+= 1 implies that t_A^+ = 1, $1 - f_A^+$= 1 and ν_A^-= -1 implies that t_A^- = -1, $-1 - f_A^-$ = -1 for all x ∈ U is called unit bipolar vague set and it is denoted by 1.

Definition 2.8: [4] Let A = ⟨x, $[t_A^+, 1 - f_A^+], [-1 - f_A^-, t_A^-]$⟩ and ⟨x, $[t_B^+, 1 - f_B^+], [-1 - f_B^-, t_B^-]$⟩ be two bipolar vague sets then their union, intersection and complement are defined as follows:

(i) A ∪ B = $\left\{\langle x, [t_{A\cup B}^+(x), 1 - f_{A\cup B}^+(x)], \frac{[-1 - f_{A\cup B}^-(x), t_{A\cup B}^-(x)]\rangle}{x} \in X\right\}$ where

$t_{A\cup B}^+(x)$ = max $\{t_A^+(x), t_B^+(x)\}$, $t_{A\cup B}^-(x)$ = min $\{t_A^-(x), t_B^-(x)\}$ and

$1 - f_{A\cup B}^+(x)$ = max $\{1 - f_A^+(x), 1 - f_B^+(x)\}$,

$-1 - f_{A\cup B}^-(x)$ = min $\{-1 - f_A^-(x), -1 - f_B^-(x)\}$.

(ii) A ∩ B = $\left\{\langle x, [t_{A\cap B}^+(x), 1 - f_{A\cap B}^+(x)], \frac{[-1 - f_{A\cap B}^-(x), t_{A\cap B}^-(x)]\rangle}{x} \in X\right\}$ where

$t_{A\cap B}^+(x)$ = min $\{t_A^+(x), t_B^+(x)\}$, $t_{A\cap B}^-(x)$ = max $\{t_A^-(x), t_B^-(x)\}$ and

$1 - f_{A\cap B}^+(x)$ = min $\{1 - f_A^+(x), 1 - f_B^+(x)\}$,

$-1 - f_{A\cup B}^-(x)$ = max $\{-1 - f_A^-(x), -1 - f_B^-(x)\}$.

(iii) A^c = {⟨x, $[f_A^+(x), 1 - t_A^+(x)], [-1 - t_A^-(x), f_A^-(x)]$⟩/ x ∈ X}.

Definition 2.9: [4] Let A and B be two bipolar vague sets defined over a universe of discourse X. We say that A ⊆ B if and only if $t_A^+(x) \le t_B^+(x)$, $1 - f_A^+(x) \le 1 - f_B^+(x)$ and $t_A^-(x) \ge t_B^-(x)$, $-1 - f_A^-(x) \ge 1 - f_B^-(x)$ for all x ∈ X.

Definition 2.10: [4] A bipolar vague topology (BVT) on a non-empty set X is a family BV_τ of bipolar vague set in X satisfying the following axioms:

(i) $0_\sim, 1_\sim$ ∈ BV_τ

(ii) $G_1 \cap G_2$ ∈ BV_τ, for any G_1, G_2 ∈ BV_τ

(iii) ∪ G_i ∈ BV_τ, for any arbitrary family $\{G_i: G_i$ ∈ BV_τ, I ∈ I$\}$.

In this case the pair (X, BV_τ) is called a bipolar vague topological space and any bipolar vague set (BVS) in BV_τ is known as bipolar vague open set in X. The complement A^c of a bipolar vague open set (BVOS) A in a bipolar vague topological space (X, BV_τ) is called a bipolar vague closed set (BVCS) in X.

Definition 2.11: [4] Let (X, BV_τ) be a bipolar vague topological space A = ⟨x, $[t_A^+, 1 - f_A^+], [-1 - f_A^-, t_A^-]$⟩ be a bipolar vague set in X. Then the bipolar vague interior and bipolar vague closure of A are defined by,

BVInt(A) = ∪ {G: G is a bipolar vague open set in X and G ⊆ A},

BVCl(A) = ∩ {K: K is a bipolar vague closed set in X and A⊆ K}.

Note that BVCl(A) is a bipolar vague closed set and BVInt(A) is a bipolar vague open set in X. Further,

(i) A is a bipolar vague closed set in X if and only if BVCl(A) = A,

(ii) A is a bipolar vague open set in X if and only if BVInt(A) = A.

Definition 2.12: [4] Let (X, BV_τ) be a bipolar vague topological space. A bipolar vague set A in (X, BV_τ) is said to be a generalized bipolar vague closed set if BVCl(A) ⊆ G whenever A⊆ G and G is bipolar vague open. The complement of a generalized bipolar vague closed set is generalized bipolar vague open set.

Definition 2.13: [4] Let (X, BV_τ) be a bipolar vague topological space and A be a bipolar vague set in X. Then the generalized bipolar vague closure and generalized bipolar vague interior of A are defined by,

GBVCl(A) = ∩ {G: G is a generalized bipolar vague closed set in X and A⊆ G},

GBInt(A) = ∪ {G: G is a generalized bipolar vague open set in X and A ⊇ G}.

Definition 2.14: [13] A bipolar vague set A of a bipolar vague topological space X, is said to be

(i) a bipolar vague α-open set if A ⊆ BVInt(BVCl(BVInt(A)))

(ii) a bipolar vague pre-open set if A ⊆ BVInt(BVCl(A))

(iii)a bipolar vague semi-open set if A ⊆ BVCl(BVInt(A))

(iv)a bipolar vague semi-α-open set if A ⊆ BVCl(αBVInt(A))

(v) a bipolar vague regular-open set BVInt(BVCl(A)) = A

(vi)a bipolar vague β-open set A ⊆ BVCl(BVInt(BVCl(A))).

Definition 2.15: [13] A bipolar vague set A of a bipolar vague topological space X, is said to be

(i) a bipolar vague α-closed set if BVCl(BVInt(BVCl(A))) ⊆ A

(ii) a bipolar vague pre-closed set if BVCl(BVInt(A)) ⊆ A

(iii)a bipolar vague semi-closed set if BVInt(BVCl(A)) ⊆ A

(iv)a bipolar vague semi-α-closed set if BVInt(αBVCl(A)) ⊆ A

(v) a bipolar vague regular-closed set if BVCl(BVInt(A)) = A

(vi)a bipolar vague β-closed set if BVInt(BVCl(BVInt(A))) ⊆ A.

Definition 2.16: [13] Let A be a bipolar vague set of a bipolar vague topological space (X, BV_τ). Then the bipolar vague α interior and bipolar vague α closure are defined as

BV_αInt(A) = ∪ {G: G is a bipolar vague α-open set in X and G ⊆ A},

BV_αCl(A) = ∩ {K: K is a bipolar vague α-closed set in X and A⊆ K}.

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Definition 2.17: [13] A bipolar vague set A in a bipolar vague topological space X, is said to be a bipolar vague α generalized closed set if BV_αCl(A) ⊆ U whenever A⊆ U and U is a bipolar vague open set in X. The complement A^c of a bipolar vague α generalized closed set A is a bipolar vague α generalized open set in X.

Definition 2.18: [4] Let (X, BV_τ) and (Y, BV_σ) be two bipolar vague topological spaces and φ : X→ Y be a function. Then φ is said to be bipolar vague continuous if and only if the preimage of each bipolar vague open set in Y is a bipolar vague open set in X.

Definition 2.19: [4] A map f : (X, BV_τ) →(Y, BV_σ) is said to be generalized bipolar vague continuous if the inverse image of every bipolar vague open set in (Y, BV_σ) is a generalized vague open set in (X, BV_τ).

Definition 2.20: [4] Let f be a mapping from a bipolar vague topological space (X, BV_τ) into a bipolar vague topological space (Y, BV_σ). Then f is said to be a bipolar vague generalized irresolute mapping if the inverse image of every bipolar vague generalized closed set in (Y, BV_σ) is a bipolar vague generalized closed set in (X, BV_τ).

Definition 2.21: [14] Let (X, BV_τ) and (Y, BV_σ) be two bipolar vague topological spaces. Then the mapping f : (X, BV_τ) →(Y, BV_σ) is called

(i) a bipolar vague α continuous if the inverse image of every bipolar vague closed set in (Y, BV_σ) is a bipolar vague α-closed set in (X, BV_τ).

(ii) a bipolar vague pre continuous if the inverse image of every bipolar vague closed set in (Y, BV_σ) is a bipolar vague pre-closed set in (X, BV_τ).

(iii) a bipolar vague semi continuous if the inverse image of every bipolar vague closed set in (Y, BV_σ) is a bipolar vague semi-closed set in (X, BV_τ).

Definition 2.22: [14] Let (X, BV_τ) and (Y, BV_σ) be two bipolar vague topological spaces. A mapping f : (X, BV_τ) →(Y, BV_σ) is called a bipolar vague α generalized continuous mapping if f^{-1}(B) is a bipolar vague α generalized closed set in (X, BV_τ) for every bipolar vague closed set B of (Y, BV_σ).

Definition 2.23: [14] A mapping f : (X, BV_τ) →(Y, BV_σ) is called a bipolar vague α generalized irresolute mapping if f^{-1}(A) is a bipolar vague α generalized closed set in (X, BV_τ) for every bipolar vague α generalized closed set A of (Y, BV_σ).

Definition 2.24: [14] A bipolar vague topological space (X, BV_τ) is said to be bipolar vague αa $T_{1/2}$(B$V_{\alpha a}T_{1/2}$) space if every bipolar vague α generalized closed set in X is a bipolar vague closed set in X.

Definition 2.25: [14] A bipolar vague topological space (X, BV_τ) is said to be bipolar vague αb $T_{1/2}$(B$V_{\alpha b}T_{1/2}$) space if every bipolar vague α generalized closed set in X is a bipolar vague generalized closed set in X.

3. Bipolar Vague α Generalized Connectedness in Topological Spaces

In this section we have introduced bipolar vague α generalized connected space, bipolar vague α generalized super connected space and bipolar vague α generalized extremally disconnected space and we have investigated some of their properties.

Definition 3.1: A bipolar vague topological space (X, BV_τ) is said to be a bipolar vague C5-connected space if the only bipolar vague sets which are both a bipolar vague open and a bipolar vague closed set are $0_\sim$ and $1_\sim$.

Definition 3.2: A bipolar vague topological space (X, BV_τ) is said to be a bipolar vague GO-connected space if the only bipolar vague sets which are both a bipolar vague generalized open set and a bipolar vague generalized closed set are $0_\sim$ and $1_\sim$.

Definition 3.3: A bipolar vague topological space (X, BV_τ) is said to be a bipolar vague α generalized connected space if the only bipolar vague sets which are both bipolar vague α generalized open set and bipolar vague α generalized closed set are $0_\sim$ and $1_\sim$.

Proposition 3.4: Every bipolar vague α generalized connected space is a bipolar vague C5-connected space but not conversely in general.

Proof: Let (X, BV_τ) be a bipolar vague α generalized connected space. Suppose (X, BV_τ) is not a bipolar vague C5-connected space, then there exists a proper bipolar vague set A which is both bipolar vague open and bipolar vague closed in (X, BV_τ). That is A is both a bipolar vague α generalized open set and a bipolar vague α generalized closed set in (X, BV_τ). This implies that (X, BV_τ) is not a bipolar vague α generalized connected space. This is a contradiction. Therefore (X, BV_τ) is a bipolar vague C5-connected space.

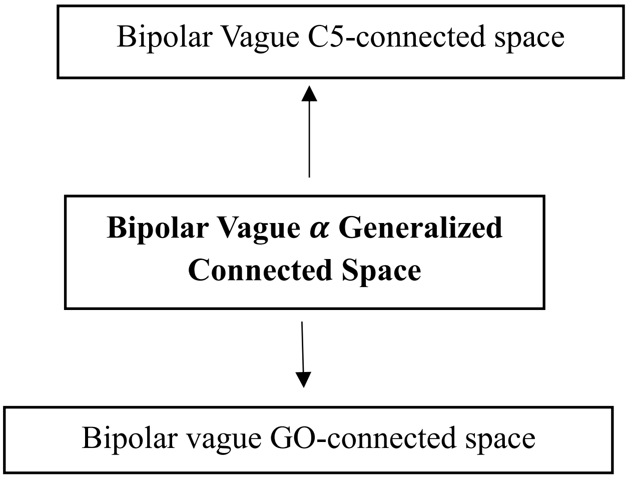
Example 3.5: Let X = {a, b} and $\tau = \{0_\sim, A, B, 1_\sim\}$ where A = ⟨x, [0.5, 0.6] [-0.6, -0.6], [0.6, 0.9] [-0.6, -0.5]⟩ and B = ⟨x, [0.4, 0.5] [-0.5, -0.5], [0.4, 0.6] [-0.5, -0.4]⟩. Then τ is a bipolar vague topology. Let M = ⟨x, [0.5, 0.6] [-0.5, -0.4], [0.4, 0.5] [-0.4, -0.3]⟩ be any bipolar vague set in X. Here (X, BV_τ) is a bipolar vague C5-connected space but not a bipolar vague α generalized connected space, since the bipolar vague set M = ⟨x, [0.5, 0.6] [-0.5, -0.4], [0.4, 0.5] [-0.4, -0.3]⟩ is both a bipolar vague α generalized open and a bipolar vague α generalized closed set in (X, BV_τ) as M ⊆ A where A is a bipolar vague open set in X. Now $BV_\alpha Cl(M) = M \cup B^c = B^c \subseteq A$.

Proposition 3.6: Every bipolar vague α generalized connected space is a bipolar vague GO-connected space but not conversely in general.

Proof: Let (X, BV_τ) be a bipolar vague α generalized connected space. Suppose (X, BV_τ) is not a bipolar vague GO-connected space, then there exists a proper bipolar vague set A which is both a bipolar vague generalized open set and a bipolar vague generalized closed set in (X, BV_τ). That is A is both a bipolar vague α generalized open set and a bipolar vague α generalized closed set in (X, BV_τ). This implies that (X, BV_τ) is not a bipolar vague α generalized connected space. This is a contradiction to our hypothesis. Therefore (X, BV_τ) is a bipolar vague GO-connected space.

Example 3.7: Let X = {a, b} and $\tau = \{0_\sim, A, B, 1_\sim\}$ where A = ⟨x, [0.5, 0.6] [-0.6, -0.6], [0.6, 0.9] [-0.6, -0.5]⟩ and B = ⟨x, [0.4, 0.5] [-0.5, -0.5], [0.4, 0.6] [-0.5, -0.4]⟩. Then τ is a bipolar vague topology. Let M = ⟨x, [0.5, 0.6] [-0.5, -0.4], [0.4, 0.5] [-0.4, -0.3]⟩ be any bipolar vague set in X. Here (X, BV_τ) is a bipolar vague GO-connected space but not a bipolar vague α generalized connected space, since the bipolar vague set M = ⟨x, [0.5, 0.6] [-0.5, -0.4], [0.4, 0.5] [-0.4, -0.3]⟩ is both a bipolar vague α generalized open and a bipolar vague α generalized closed set in (X, BV_τ) as $M \subseteq A$ where A is a bipolar vague open set in X. Now $BV_\alpha Cl(M) = M \cup B^c = B^c \subseteq A$.

The relation between various types of bipolar vague connectedness is given in the following diagram:



Proposition 3.8: A bipolar vague topological space (X, BV_τ) is a bipolar vague α generalized connected space if and only if there exists no non-zero bipolar vague α generalized open sets A and B in (X, BV_τ) such that $A = B^c$.

Proof:

Necessity: Let A and B be two bipolar vague α generalized open sets in (X, BV_τ) such that $A \neq 0_\sim$, $B \neq 0_\sim$ and $A = B^c$. Therefore, B^c is a bipolar vague α generalized closed set. Since $B \neq 0_\sim$, $A = B^c \neq 1_\sim$. This implies A is a proper bipolar vague set which is both bipolar vague α generalized open set and a bipolar vague α generalized closed set in (X, BV_τ). Hence (X, BV_τ) is not a bipolar vague α generalized connected space. But this is a contradiction to our hypothesis. Thus, there exist no non-zero bipolar vague α generalized open sets A and B in (X, BV_τ) such that $A = B^c$.

Sufficiency: Let A be both a bipolar vague α generalized open set and bipolar vague α generalized closed set in (X, BV_τ) such that $1_\sim \neq A \neq 0_\sim$. Now let $B = A^c$. Then B is a bipolar vague α generalized open set and $B \neq 1_\sim$. This implies $B^c = A \neq 0_\sim$, which is a contradiction to our hypothesis. Therefore (X, BV_τ) is a bipolar vague α generalized connected space.

Proposition 3.9: If f : (X, BV_τ) → (Y, BV_σ) is a bipolar vague α generalized continuous mapping and (X, BV_τ) is a bipolar vague α generalized connected space, then (Y, BV_σ) is a bipolar vague C5-connected space.

Proof: Let (X, BV_τ) be a bipolar vague α generalized connected space. Suppose (Y, BV_σ) is not a bipolar vague C5-connected space, then there exists a proper bipolar vague set A which is both bipolar vague open set and bipolar vague closed set in (Y, BV_σ). Since f is a bipolar vague α generalized continuous mapping, f^{-1}(A) is both a bipolar vague α generalized open set and a bipolar vague α generalized closed set in (X, BV_τ). But this is a contradiction to hypothesis. Hence (Y, BV_σ) is a bipolar vague C5-connected space.

Proposition 3.10: If f : (X, BV_τ) → (Y, BV_σ) is a bipolar vague α generalized irresolute surjection mapping and (X, BV_τ) is a bipolar vague α generalized connected space, then (Y, BV_σ) is also a bipolar vague α generalized connected space.

Proof: Suppose (Y, BV_σ) is not a bipolar vague α generalized connected space, then there exists a proper bipolar vague set B which is both bipolar vague α generalized open set and bipolar vague α generalized closed set in (Y, BV_σ). Since f is a bipolar vague α generalized irresolute mapping, f^{-1}(B) is both a bipolar vague α generalized open set and bipolar vague α generalized closed set in (X, BV_τ). But this is a contradiction to hypothesis. Hence (Y, BV_σ) is a bipolar vague α generalized connected space.

Proposition 3.11: A bipolar vague topological space (X, BV_τ) is a bipolar vague α generalized connected space if and only if there exists no non-zero bipolar vague α generalized open sets A and B in (X, BV_τ) such that B = A^c, B = (BV_αCl(A))^c and A = (BV_αCl(B))^c .

Proof:

Necessity: Assume that there exist bipolar vague sets A and B such that A ≠ $0_\sim$ ≠ B, B = A^c, B = (BV_αCl(A))^c and A = (BV_αCl(B))^c . Since (BV_αCl(A))^c and (BV_αCl(B))^c are bipolar vague α generalized open sets in (X, BV_τ), A and B are bipolar vague α generalized open sets in (X, BV_τ). This implies (X, BV_τ) is not a bipolar vague α generalized connected space, which is a contradiction. Therefore, there exists no non-zero bipolar vague α generalized open sets A and B in (X, BV_τ) such that B = A^c, B = (BV_αCl(A))^c and A = (BV_αCl(B))^c .

Sufficiency: Let A be both a bipolar vague α generalized open set and bipolar vague α generalized closed set in (X, BV_τ) such that $1_\sim$ ≠ A ≠ $0_\sim$. Now by taking B = A^c, we obtain a contradiction to our hypothesis. Hence (X, BV_τ) is a bipolar vague α generalized connected space.

Proposition 3.12: Let (X, BV_τ) be a B$V_{\alpha a}T_{1/2}$ space, then the following are equivalent.

(i) (X, BV_τ) is a bipolar vague α generalized connected space.

(ii) (X, BV_τ) is a bipolar vague GO-connected space.

(iii) (X, BV_τ) is a bipolar vague C5-connected space.

Proof: (i) ⇒(ii) is obvious from Proposition 3.6.

(ii) ⇒ (iii) is obvious.

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(iii) ⇒ (i) Let (X, BV_τ) be a bipolar vague C5-connected space. Suppose (X, BV_τ) is not a bipolar vague α generalized connected space, then there exists a proper bipolar vague set A in (X, BV_τ) which is both a bipolar vague α generalized open set and a bipolar vague α generalized closed set in (X, BV_τ). But since (X, BV_τ) is a $BV_{\alpha a}T_{1/2}$ space, A is both bipolar vague open set and bipolar vague closed set in (X, BV_τ). This implies that (X, BV_τ) is not a bipolar vague C5-connected space, which is a contradiction to our hypothesis. Therefore (X, BV_τ) is a bipolar vague α generalized connected space.

Definition 3.13: Two bipolar vague sets A and B are said to be q-coincident (AqB in short) if and only if there exits an element $x \in$ X such that $\nu_A^+(x) > \nu_B^-(x)$ or $\nu_A^-(x) < \nu_B^+(x)$.

Definition 3.14: A bipolar vague topological space (X, BV_τ) is a bipolar vague C5-connected between two bipolar vague sets A and B if there is no bipolar vague open set E in (X, BV_τ) such that A ⊆ E and Eq^cB.

Definition 3.15: A bipolar vague topological space (X, BV_τ) is called bipolar vague α generalized connected between two bipolar vague sets A and B if there is no bipolar vague α generalized open set E in (X, BV_τ) such that A ⊆ E and Eq^cB.

Example 3.16: Let X = {a, b} and let $\tau = \{0_\sim, A, 1_\sim\}$ be a bipolar vague topology on X, where A = ⟨x, [0.5, 0.5] [-0.5, -0.5], [0.4, 0.4] [-0.4, -0.4]⟩.Then bipolar vague topological space (X, BV_τ) is a bipolar vague α generalized connected between the two bipolar vague sets M = ⟨x, [0.3, 0.3] [-0.3, -0.3], [0.2, 0.2] [-0.2, -0.2]⟩ and N = ⟨x, [0.2, 0.2] [-0.2, -0.2], [0.2, 0.2] [-0.2, -0.2]⟩ as there exists no bipolar vague α generalized open set E such that M ⊆ E and Eq^cN.

Proposition 3.17: If a bipolar vague topological space (X, BV_τ) is a bipolar vague α generalized connected between the two bipolar vague sets A and B, then it is a bipolar vague C5-connected between two bipolar vague sets A and B but the converse my not be true in general.

Proof: Suppose (X, BV_τ) is not a bipolar vague C5-connected between A and B, then there exists a bipolar vague open set E in (X, BV_τ) such that A ⊆ E and Eq^cB. Since every bipolar vague open set is a bipolar vague α generalized open set, there exists a bipolar vague α generalized open set E in (X, BV_τ) such that A ⊆ E and Eq^cB. This implies (X, BV_τ) is not a bipolar vague α generalized connected between A and B, a contradiction to our hypothesis. Therefore (X, BV_τ) is a bipolar vague C5-connected between A and B.

Example 3.18: Let X = {a, b} and let $\tau = \{0_\sim, A, 1_\sim\}$ be a bipolar vague topology on X, where A = ⟨x, [0.5, 0.4] [-0.4, -0.5], [0.5, 0.6] [-0.6, -0.5]⟩.Then bipolar vague topological space (X, BV_τ) is a bipolar vague C5-connected between the two bipolar vague sets M = ⟨x, [0.3, 0.2] [-0.2, -0.3], [0.7, 0.6] [-0.6, -0.7]⟩ and N = ⟨x, [0.6, 0.6] [-0.2, -0.2], [0.2, 0.2] [-0.2, -0.2]⟩. But (X, BV_τ) is not a bipolar vague α generalized connected between M and N, since the bipolar vague set E = ⟨x, [0.3, 0.3] [-0.3, -0.3], [0.7, 0.7] [-0.7, -0.7]⟩ is a bipolar vague α generalized open set in X such that M ⊆ E and E ⊆ N^c.

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Proposition 3.19: A bipolar vague topological space (X, BV_τ) is a bipolar vague α generalized connected between two bipolar vague sets A and B if and only if there is no bipolar vague α generalized open set and bipolar vague α generalized closed set E in (X, BV_τ) such that $A \subseteq E \subseteq B^c$.

Proof:

Necessity: Let (X, BV_τ) be a bipolar vague α generalized connected between two bipolar vague sets A and B. Suppose that there exists a bipolar vague α generalized open set and a bipolar vague α generalized closed set E in (X, BV_τ) such that $A \subseteq E \subseteq B^c$, then Eq^cB and $A \subseteq E$. This implies (X, BV_τ) is not a bipolar vague α generalized connected between A and B, by Definition 3.15. It is a contradiction to our hypothesis. Therefore, there is no bipolar vague α generalized open set and bipolar vague α generalized closed set E in (X, BV_τ) such that $A \subseteq E \subseteq B^c$.

Sufficiency: Suppose that (X, BV_τ) is not a bipolar vague α generalized connected between A and B. Then there exists a bipolar vague α generalized open set E in (X, BV_τ) such that $A \subseteq E$ and Eq^cB. This implies that there is no bipolar vague α generalized open set E in (X, BV_τ) such that $A \subseteq E \subseteq B^c$. But this is a contradiction to our hypothesis. Hence (X, BV_τ) is a bipolar vague α generalized connected between A and B.

Proposition 3.20: If a bipolar vague topological space (X, BV_τ) is a bipolar vague α generalized connected between two bipolar vague sets A and B, $A \subseteq A_1$ and $B \subseteq B_1$, then (X, BV_τ) is a bipolar vague α generalized connected between A_1 and B_1.

Proof: Suppose that (X, BV_τ) is not a bipolar vague α generalized connected between A_1 and B_1 , then by Definition 3.15, there exists a bipolar vague α generalized open set E in (X, BV_τ) such that $A_1 \subseteq E$ and Eq^cB_1. This implies $E \subseteq B_1{}^c$ and $A_1 \subseteq E$ implies $A \subseteq A_1 \subseteq E$. Hence $A \subseteq E$. Since $E \subseteq B_1{}^c$, $B_1 \subseteq E^c$, $B \subseteq B_1 \subseteq E^c$. Hence $E \subseteq B^c$. Therefore (X, BV_τ) is not a bipolar vague α generalized connected between A and B, which is a contradiction to our hypothesis. Thus (X, BV_τ) is a bipolar vague α generalized connected between A_1 and B_1.

Proposition 3.21: Let (X, BV_τ) be a bipolar vague topological space and A and B be a bipolar vague sets in (X, BV_τ). If AqB, then (X, BV_τ) is a bipolar vague α generalized connected between A and B.

Proof: Suppose (X, BV_τ) is not a bipolar vague α generalized connected between A and B. Then there exists a bipolar vague α generalized open set E in (X, BV_τ) such that $A \subseteq E$ and $E \subseteq B^c$. This implies that $A \subseteq B^c$. That is Aq^cB. But this is a contradiction to our hypothesis. Therefore (X, BV_τ) is a bipolar vague α generalized connected between A and B.

Definition 3.22: A bipolar vague set A is called a bipolar vague regular α generalized open set if A = BV_αInt(BV_αCl(A)). The complement of a bipolar vague regular α generalized open set is called a bipolar vague regular α generalized closed set.

Definition 3.23: A bipolar vague topological space (X, BV_τ) is called a bipolar vague α generalized super connected space if there exists no proper bipolar vague regular α generalized open set in (X, BV_τ).

Proposition 3.24: Let (X, BV_τ) be a bipolar vague topological space, then the following are equivalent:

(i) (X, BV_τ) is a bipolar vague α generalized super connected space

(ii) For every non-zero bipolar vague regular α generalized open set A, $BV_\alpha Cl(A) = 1_\sim$

(iii) For every bipolar vague regular α generalized closed set A with $A \neq 1_\sim$, $BV_\alpha Int(A) = 0_\sim$

(iv) There exists no bipolar vague regular α generalized open sets A and B in (X, BV_τ) such that $A \neq 0_\sim \neq B$, $A \subseteq B^c$

(v) There exists no bipolar vague regular α generalized open sets A and B in (X, BV_τ) such that $A \neq 0_\sim \neq B$, $B = (BV_\alpha Cl(A))^c$, $A = (BV_\alpha Cl(B))^c$

(vi) There exists no bipolar vague regular α generalized closed sets A and B in (X, BV_τ) such that $A \neq 1_\sim \neq B$, $B = (BV_\alpha Int(A))^c$, $A = (BV_\alpha Int(B))^c$

Proof:

(i)⇒(ii) Assume that there exists a bipolar vague regular α generalized open set A in (X, BV_τ) such that $A \neq 0_\sim$ and $BV_\alpha Cl(A) \neq 1_\sim$. Now let $B = BV_\alpha Int(BV_\alpha Cl(A))^c$. Then B is a proper bipolar vague regular α generalized open sets in (X, BV_τ). But this is a contradiction to the fact that (X, BV_τ) is a bipolar vague regular α generalized super connected space. Therefore $BV_\alpha Cl(A) = 1_\sim$.

(ii) ⇒ (iii) Let $A \neq 1_\sim$ be a bipolar vague regular α generalized closed set in (X, BV_τ). If $B = A^c$, then B is a bipolar vague regular α generalized open set in (X, BV_τ) with $B \neq 0_\sim$. Hence $BV_\alpha Cl(B) \neq 1_\sim$, by hypothesis. This implies $(BV_\alpha Cl(B))^c = 0_\sim$. That is $BV_\alpha Int(B^c) = 0_\sim$. Hence $BV_\alpha Int(A) = 0_\sim$.

(iii) ⇒ (iv) Suppose A and B be two bipolar vague regular α generalized open sets in (X, BV_τ) such that $A \neq 0_\sim \neq B$, $A \subseteq B^c$. Since B^c is a bipolar vague regular α generalized closed set in (X, BV_τ) and $B \neq 0_\sim$ implies $B^c \neq 1_\sim$, $B^c = BV_\alpha Cl(BV_\alpha Int(B^c))$ and we have $BV_\alpha Int(B^c) = 0_\sim$. But $A \subseteq B^c$. Therefore $0_\sim \neq A = BV_\alpha Int(BV_\alpha Cl(A)) \subseteq BV_\alpha Int(BV_\alpha Cl(B^c)) = BV_\alpha Int(BV_\alpha Cl(BV_\alpha Cl(BV_\alpha Int(B^c)))) = BV_\alpha Int(BV_\alpha Cl(BV_\alpha Int B^c))) = BV_\alpha Int(B^c) = 0_\sim$ which is a contradiction. Therefore (iv) is true.

(iv)⇒(i) Suppose $0_\sim \neq A \neq 1_\sim$ be a bipolar vague regular α generalized open set in (X, BV_τ). If we take $B = (BV_\alpha Cl(A))^c$, then B is a bipolar vague regular α generalized open set, since $BV_\alpha Int(BV_\alpha Cl(B)) = BV_\alpha Int(BV_\alpha Cl(BV_\alpha Cl(A))^c) = BV_\alpha Int(BV_\alpha Int(BV_\alpha Cl(A)))^c = BV_\alpha Int(A^c) = (BV_\alpha Cl(A))^c = B$. Also, we get $B \neq 0_\sim$, since otherwise, if $B = 0_\sim$, this implies $(BV_\alpha Cl(A))^c = 0_\sim$. That is $BV_\alpha Cl(A) = 1_\sim$. Hence $A = BV_\alpha Int(BV_\alpha Cl(A)) = BV_\alpha Int(1_\sim) = 1_\sim$, which is a contradiction. Therefore $B \neq 0_\sim$ and $A \subseteq B^c$. But this is a contradiction to (iv). Therefore (X, BV_τ) is a bipolar vague α generalized super connected space.

(i) ⇒ (v) Suppose A and B are any two bipolar vague regular α generalized open sets in (X, BV_τ) such that $A \neq 0_\sim \neq B$, $B = (BV_\alpha Cl(A))^c$ and $A = (BV_\alpha Cl(B))^c$. Now we have $BV_\alpha Int(BV_\alpha Cl(A)) = BV_\alpha Int(B^c) = (BV_\alpha Cl(B))^c = A$, $A \neq 0_\sim$ and $A \neq 1_\sim$, since if $A \neq 1_\sim$, then $1_\sim = (BV_\alpha Cl(B))^c \Rightarrow BV_\alpha Cl(B) = 0_\sim \Rightarrow B = 0_\sim$. But $B \neq 0_\sim$. Therefore $A \neq 1_\sim \Rightarrow A$ is a proper bipolar vague regular α generalized open set in (X, BV_τ), which is a contradiction to (i). Hence (v) is true.

(v) ⇒ (i) Suppose A is a bipolar vague regular α generalized open set in (X, BV_τ) such that $0_\sim \neq A \neq 1_\sim$. Now take $B = (BV_\alpha Cl(A))^c$. In this case we get $B \neq 0_\sim$ and B is a bipolar vague regular α generalized open set in (X, BV_τ), $B = (BV_\alpha Cl(A))^c$ and $(BV_\alpha Cl(B))^c = (BV_\alpha Cl(BV_\alpha Cl(A))^c)^c = BV_\alpha Int(BV_\alpha Cl(A)^c)^c =$

BV_αInt(BV_αCl (A)) = A. But this is a contradiction to (v). Therefore (X, BV_τ) is a bipolar vague α generalized super connected space.

(v) ⇒ (vi) Suppose A and B be two bipolar vague regular α generalized closed sets in (X, BV_τ) such that A ≠ $1_\sim$ ≠ B, B = (BV_αInt (A))^c and A = (BV_αInt (B))^c. Taking C = A^c and D = B^c, C and D become bipolar vague regular α generalized open sets in (X, BV_τ) with C ≠ $0_\sim$ ≠ D, D = (BV_αCl (C))^c = (BV_αCl(D))^c, which is a contradiction to (v). Hence (vi) is true.

(vi) ⇒ (v) It can be proved easily by the similar way as in (v) ⇒ (vi).

Definition 3.27: A bipolar vague topological space (X, BV_τ) is said to be a bipolar vague α generalized extremally disconnected space if the bipolar vague α generalized closure of every bipolar vague α generalized open set in (X, BV_τ) is a bipolar vague α generalized open set.

Theorem 3.28: Let (X, BV_τ) be a bipolar vague topological space, then the following are equivalent:

(i) (X, BV_τ) is a bipolar vague α generalized extremally disconnected space.

(ii) For each bipolar vague α generalized closed set A, BV_αInt(A) is a bipolar vague α generalized closed set.

(iii) For each bipolar vague α generalized open set A, BV_αCl(A) = (BV_αCl(BV_αCl(A))^c)^c

(iv) For each bipolar vague α generalized open sets A and B with BV_αCl(A) = B^c, BV_αCl(A) = (BV_αCl(B))^c.

Proof: (i) ⇒ (ii) Let A be any bipolar vague α generalized closed set. Then A^c is a bipolar vague α generalized open set. So (i) implies that BV_αCl(A^c) = (BV_αInt(A))^c is a bipolar vague α generalized open set. Then BV_αInt(A) is a bipolar vague α generalized closed set in (X, BV_τ).

(ii) ⇒ (iii) Let A be bipolar vague α generalized open set. Then we have BV_αCl(BV_αCl(A))^c = BV_αCl(BV_αInt(A^c)). Therefore (BV_αCl(BV_αCl(A))^c)^c = (BV_αCl(BV_αInt(A^c)))^c. Since A is a bipolar vague α generalized open set. Then A^c is a bipolar vague α generalized closed set. So by (ii) BV_αInt(A^c) is a bipolar vague α generalized closed set. That is BV_αCl(BV_αInt(A^c)) = BV_αInt(A^c). Hence (BV_αCl(BV_αInt(A^c)))^c = (BV_αInt(A^c))^c = BV_αCl(A).

(iii) ⇒ (iv) Let A and B be any two bipolar vague α generalized open sets in (X, BV_τ) such that BV_αCl(A) = B^c. (iii) implies BV_αCl(A) = (BV_αCl(BV_αCl(A))^c)^c = (BV_αCl(B^c))^c = (BV_αCl(B))^c.

(iv) ⇒ (i) Let A and B be any two bipolar vague α generalized open sets in (X, BV_τ) with BV_αCl(A) = B^c and BV_αCl(A) = (BV_αCl(B))^c. From BV_αCl(A) = B^c ⇒ B = (BV_αCl(A))^c. Since BV_αCl(B) is a bipolar vague α generalized closed set, this implies that BV_αCl(A) is a bipolar vague α generalized open set. This implies that (X, BV_τ) is a bipolar vague α generalized extremally disconnected space.

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