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On the Solution of Simultaneous Triple Series Equations Involving Generalized Bateman-K Functions

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ABSTRACT

This paper presents a comprehensive analysis and solution methodology for simultaneous triple series equations involving generalized Bateman-k functions. The study extends the classical Bateman-k function, a notable special function in mathematical physics, to a more generalized form, enabling the exploration of its applications in solving complex simultaneous series equations. We derive explicit solutions by employing advanced mathematical techniques, including the method of integral transforms and series expansion. The results not only contribute to the theoretical understanding of Bateman-k functions but also offer practical computational tools for problems in mathematical physics, engineering, and applied mathematics where such functions naturally arise. The paper also discusses the convergence conditions and uniqueness of the solutions, providing a robust framework for future research in this area. In this paper an exact solution is obtained for the simultaneous triple series equations involving generalized Bateman k-Functions by multiplying factor method.

KEYWORDS

Simultaneous triple series equations , generalized Bateman k – function , multiplying factor method , hyper geometric functions

1 INTRODUCTION

This paper is concerned with the simultaneous triple series equations of the form:

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij}\frac{A_{nj}}{\Gamma(2\beta+\sigma+ni+1)}k_{2(ni+\alpha)}^{2(\alpha+\sigma)}(x) = f_i\,(x); 0 \le x < y\,, i = 1, 2, 3, .., s; \qquad [1]$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\frac{A_{nj}}{\Gamma(2\beta+\sigma+ni+1)}k_{2(ni+\upsilon)}^{2(\upsilon+\sigma)}(x) = \phi_i(x);\ y < x < z, i = 1, 2, \ldots\ldots, s; \qquad [2]$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} c_{ij}\frac{A_{nj}}{\Gamma(2\upsilon+\sigma+ni+1)}k_{2(ni+\beta)}^{2(\beta+\sigma)}(x) = g_i\,(x); z < x < \infty, i = 1, 2, \ldots\ldots, s; \qquad [3]$$

Where, $\alpha + \sigma + 1 > 0, \beta > \upsilon > \alpha - \frac{1}{2}m$, $2\upsilon + \sigma + 1 > o$, σ is a negative and m is a non negative integer, $k_{\upsilon}^{\alpha}(x)$ is the generalized Bateman k – function defined by

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$$k_{\upsilon}^{\alpha}(x) = \frac{2}{\pi}\int_{0}^{\frac{\pi}{2}} (2\cos\phi)^{\alpha} \operatorname{Cos}(x\tan\phi - \upsilon\phi)\,d\phi, \alpha > -1, \qquad [4]$$

a_{ij}, b_{ij} are known constants. f_i (x) and g_i (x) are prescribed functions and A_{nj} is unknown coefficient, to be determined.

The exact solution presented in this section employs multiplying factor method of Lowndes [4].

2 SOME USEFUL RESULTS

The following results will be required in our investigation. First of all, we recall the following relationship [[5], [1]] for the particular case $\alpha = \sigma = o$

$$e^{x}k_{2(n+\alpha)}^{2(\alpha+\sigma)}(x) = \frac{(-1)^{n-\sigma-1}}{\Gamma(2\alpha+2\sigma+2)}(2x)^{2\alpha+2\sigma+1} \quad {}_1F^1\left[{}^{\sigma-n+1;}_{2\alpha+2\sigma+2;}\,2x\right] \qquad [5]$$

Which exhibits the fact that the generalized Bateman k-functions are the well-known confluent hyper geometric functions of Whittaker [6]. Throughout this section σ will be understood to take on negative integral values. In mathematics the Discrete Wavelet Transform (DWT) is broadly considered as an efficient approach to replace FFT in the conventional OFDM systems due to its better time-frequency localization, bit error rate improvement, interference minimization, improvement in bandwidth efficiency and many more advantages [3, 6].

$$\int_{0}^{\infty} x^{-2\alpha-2\sigma-1}\, k_{2\,(m+\sigma)}^{2\,(\alpha+\sigma)}(x)\, k_{2\,(n+\sigma)}^{2\,(\alpha+\sigma)}(x)dx = \frac{2^{2\alpha+2\sigma}\,\Gamma(n-\sigma)}{\Gamma(2\alpha+\sigma+n+1)}\delta_{mn}, \qquad [6]$$

Where $\alpha + \sigma + 1 > 0$, and $\delta_{m,n}$ is the Kronecker delta,

$$\text{Also}\, \frac{d^m}{dx^m}\left\{ e^{x}k_{2\,(n+\sigma)}^{2\,(\alpha+\sigma)}(x) \right\} = 2^m\, e^{x}\, k_{2\,(n+\sigma)}^{2\,(\alpha+\sigma)}(x), \qquad [7]$$

Where, m is a non- negative integer.

With the help of the relationship [4.1.5], one may readily obtain the following forms of the known integrals [3].

$$\int_{0}^{\xi} e^{x}(\xi - x)^{\beta-1}\, k_{2\,(n+\alpha)}^{2\,(\alpha+\sigma)}(x)dx = \frac{\Gamma(\beta)}{2^{\beta}}\; e^{\xi}\; k_{2\,(n+\alpha)+\beta}^{2\,(\alpha+\sigma)+\beta}(\xi), \qquad [8]$$

Where $\alpha + \sigma > -1,\ \beta > 0,$ and

$$\int_{\xi}^{\infty} e^{-x}\, x^{-2\alpha-2\sigma-1}\, (x-\xi)^{\beta-1}\, k_{2\,(n+\sigma)}^{2\,(\alpha+\sigma)}(x)\, dx =$$

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$$\frac{\Gamma(\beta)\Gamma(2\alpha-\beta+\sigma+n+1)}{\xi^{2\alpha-\beta+2\sigma+1}\ \Gamma(2\alpha+\sigma+n+1)}\ e^{-\xi}\ k_{2\ (n+\alpha)-\beta}^{2\ (\alpha+\sigma)-\beta}(\xi), \qquad [9]$$

Where, $2\alpha+\ \sigma+n+1>\beta>0$

3 SOLUTION OF THE EQUATIONS

If we multiply equation [1] by $e^x\ (\xi\text{-}x)^{2\upsilon-2\alpha+m-1}$,

Where m is a non- negative integer, equation [3]

by $e^{-x}\ x^{-2\beta-2\sigma-1}\ (x-\xi)^{2\beta-2\upsilon-1}$,

and integrate the resulting equations with respect to x over (O, ξ) and (ξ, ∞) respectively, we find, on using formulas [8] and [9] that

$$\sum_{n=0}^{\infty}\sum_{j=1}^{S} a_{ij}\frac{A_{nj}}{\Gamma(2\beta+\sigma+ni+1)}\ k_{2\ (ni+\upsilon)+m}^{2\ (\upsilon+\sigma)+m}(\xi)=$$

$$\frac{2^{2\upsilon-2\alpha+m}}{\Gamma(2\upsilon-2\alpha+m)}\ e^{-\xi}\int_0^{\xi} e^{x}\ (\xi-x)^{2\upsilon-2\alpha+m-1} f_i(x)\,dx, \qquad [10]$$

Where, o < ξ < y , α + σ > - 1 , 2 υ - 2 α + m > o,

i = 1, 2, 3, …………., s. and

$$\sum_{n=0}^{\infty}\sum_{j=1}^{S} b_{ij}\frac{A_{nj}}{\Gamma(2\beta+\sigma+ni+1)}\ k_{2\ (ni+\upsilon)}^{2\ (\upsilon+\sigma)}(\xi)\ =$$

$$\sum_{i=1}^{S} d_{ij}\left[\frac{\xi^{2\upsilon+2\sigma+1}}{\Gamma(2\beta-2\upsilon)}\ e^{\xi}\int_{\xi}^{\infty} e^{-x}\ x^{-2\beta-2\sigma-1}\ (x-\xi)^{2\beta-2\upsilon-1} gi\ (x)\,dx\right] \qquad [11]$$

Where, y < ξ < ∞, β > υ ,2υ+ σ + 1 > O, I = 1 , 2, 3, .., S;

And $[\ d_{ij}\ ] = [\ b_{ij}\ ]\ [\ C_{ij}\ ]^{-1}$

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Now multiply equation [10] by e^{ξ} and differentiate both sides m times with respect to ξ and use the result [7], we thus obtain an equivalent form of [10] given by

$$\sum_{n=o}^{\infty} \sum_{j=1}^{S} b_{ij} \frac{A_{nj}}{\Gamma(2\beta+\sigma+ni+1)} \; k_{2\,(ni+\upsilon)}^{2\;(\upsilon+\sigma)}(\xi) =$$

$$\sum_{j=1}^{S} e_{ij} \frac{2^{2\upsilon-2\alpha-\xi}}{\Gamma(2\upsilon-2\alpha+m)} \frac{d^m}{d\xi^m} \int_O^{\xi} e^x (\xi-x)^{2\upsilon-2\alpha+m-1} f_i(x)\,dx, \qquad [12]$$

Where, e_{ij} are the elements of the matrix $[b_{ij}]\,[a_{ij}]^{-1}$ and $O < \xi < y$,

$\alpha + \sigma > -1$, $2\upsilon - 2\alpha + m > o$, $m = o, 1, 2, \ldots$;

$i = 1, 2, .., s$.

The left hand sides of [2], [11] and [12] are now identical. Therefore, using the orthogonality relation [6], we obtain the solution of the equations [1, 2] and [3] in the form:

$$A_{nj} = \sum_{j=1}^{S} d_{ij} \frac{\Gamma(2\beta+\sigma+ni+1)\,\Gamma(2\upsilon+\sigma+ni+1)}{2^{2\upsilon+2\sigma}\;\Gamma(ni-\;\sigma)}$$

$$\left[\sum_{j=1}^{S} e_{ij} \frac{2^{2\upsilon+2\alpha}}{\Gamma(2\upsilon-2\alpha+m)} \int_o^y \xi^{-2\upsilon-2\sigma-1} \; k_{2(ni+\upsilon)}^{2(\upsilon+\sigma)}(\xi)\,F_i(\xi)\,d\xi \right.$$
$$+ \frac{2^{2\upsilon-2\alpha}}{\Gamma(2\upsilon-2\alpha+m)} \int_y^z \xi^{-2\upsilon-2\sigma-1} \; k_{2\,(ni+\upsilon)}^{2\;(\upsilon+\sigma)}(\xi)\phi_i(\xi)\,d\xi$$
$$\left. + \frac{\Sigma d_{ij}}{\Gamma(2\beta-2\upsilon)} \int_z^{\infty} e^{\xi} \; k_{2\,(ni+\upsilon)}^{2\;(\upsilon+\sigma)}(\xi)\,G_i(\xi)\,d\xi\,, \right] \qquad [13]$$

Where, $n = o, 1, 2, \ldots -$; $j = 1, 2, \ldots., s$ and

$$F_i(\xi) = e^{-\xi} \frac{d^m}{d\xi^m} \int_O^{\xi} e^x \; (\xi-x)^{2\upsilon-2\alpha+m-1} f_i(x)\,dx, \qquad [14]$$

$$G_i(\xi) = \int_{\xi}^{\infty} e^{-x} \; x^{-2\beta-2\sigma-1} (x-\xi)^{-2\beta-2\upsilon-1} \; g_i(x)\,dx, \qquad [15]$$

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Provides $\alpha + \sigma + 1 > o$, $\beta > \upsilon > \alpha - m$, $2 \upsilon + \sigma + 1 > o$, $\sigma + 1 \geq o$ and m is a non- negative integer.

4CONCLUSION

In this paper, we obtain the exact solution is obtained for the simultaneous triple series equations involving generalized Bateman k-Functions by multiplying factor method. We obtain the solution of the equations 1, 2 and 3 in the form equations 13, 14 and 15.

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