Pythagorean Neutrosophic Cubic Topology

Berna Joyce L[1] and Elvina Mary L[2]

[1]Research Scholar, [2]Assistant Professor

[1, 2]PG and Research Department of Mathematics, Nirmala College for Women, Coimbatore, Tamil Nadu, India.

Email: [1]bernajoyceslr@gmail.com [2]elvinalawrence07@gmail.com

ABSTRACT

This paper deals with Pythagorean Neutrosophic Cubic Topology. The concept of Pythagorean Neutrosophic Cubic Set apply to topological space and introduce the notation of Pythagorean Neutrosophic Topological space. Further we introduce P-Pythagorean Neutrosophic Cubic Topological Space and R-Pythagorean Neutrosophic Cubic Topological Space. We examine basic properties of Pythagorean Neutrosophic Cubic Topological space.

Keywords Pythagorean Neutrosophic Cubic Set, Pythagorean Neutrosophic Cubic Topological Space.

1. INTRODUCTION

Zadeh[1] established a foundation for fuzzy mathematics in 1965 with the development of the theory of fuzzy sets. Soon after, in 1975, Zadeh[2] proposed a reconfiguration of fuzzy sets with interval valued function membership. Neutrosophic Sets were initially defined in 1995 by Florentin Smarandache[3]. This enables ambiguity and uncertainty to be handled more thoroughly. In the year 2012, the introduction of the important theory of Cubic Sets was defined by Jun et al[4]. In 2017, Chang Su Kim, Florentin Smarandache, and Young Bae Jun[5] presented the an idea of Neutrosophic Cubic Sets. Yagar[6] first presented the Pythagorean Fuzzy set's evolution in 2013. In 2019, F. Khana, M. S. Ali Khana, M. Shahzada, and S. Abdullah[7] gave a concept of the Pythagorean Cubic Fuzzy Set. R. Jhansi & K. Mohana[8] proposed the Pythagorean Neutrosophic Sets. The interval-valued Neutrosophic Pythagorean Sets were presented by Stephy et al[9]. A combination of Pythagorean Neutrosophic Sets and Pythagorean Neutrosophic Interval Valued Sets, Berna Joyce[10] introduced a novel idea for Pythagorean Neutrosophic Cubic Sets (PNCS), including Internal Pythagorean Neutrosophic Cubic Sets (IPNCS) and External Pythagorean Neutrosophic Cubic Sets (EPNCS) and then presented some logic operations of PNCS such as P-union, P-intersection, R-union and R-intersection of PNCS[11]. In 1968, C.L. Chang[12] introduced Fuzzy Topological Space This paper portrays topological structure on Pythagorean Neutrosophic Cubic Set theory. We define Pythagorean Neutrosophic Cubic Topological Space (PNCTS) and also discuss two type of PNCTS i.e., P order of Pythagorean Neutrosophic Cubic Topological Space(P-PNCTS) and R order of Pythagorean Neutrosophic Cubic Topological Space (R-PNCTS)

2. PRELIMINARIES

Definition 2.1[10] Let X be a non-empty set of the universe. A **Pythagorean Neutrosophic Cubic Set** (PNCS) can be defined as follows $\mathbb{A} = \{\langle x, A(x), \lambda(x)\rangle : x \in X\}$ where $A(x)$ represent the Pythagorean Neutrosophic Interval valued set in X $\lambda(x)$ represent the Pythagorean Neutrosophic Set. PNCS can be denoted as a pair $\mathbb{A} = (A, \lambda)$. A PNCS $\mathbb{A} = (A, \lambda)$ in which $A(x) = 0$ and $\lambda(x) = 0$ is denoted by $\hat{0} \forall x \in X$ respectively, $A(x) = 1$ and $\lambda(x) = 1 \forall x \in X$ is denoted as $\hat{1}$.

Definition 2.2[10] Let X be a non-empty set of the universe. A Pythagorean Neutrosophic cubic set $\mathbb{A} = \{A, \lambda\}$ in X is said to be **Internal Pythagorean Neutrosophic Cubic Set** if $\left(T_A^-(x) \leq \lambda_T(x) \leq T_A^+(x)\right) \forall x \in X$, $\left(I_A^-(x) \leq \lambda_I(x) \leq I_A^+(x)\right) \forall x \in X$ and $\left(F_A^-(x) \leq \lambda_F(x) \leq F_A^+(x)\right) \forall x \in X$.

Definition 2.3 [10] Let X be a non-empty set of the universe. A Pythagorean Neutrosophic cubic set $\mathbb{A} = \{A, \lambda\}$ in X is said to be **External Pythagorean Neutrosophic Cubic Set** $\lambda_T(x) \notin \left(T_A^-(x),\ T_A^-(x)\right) \forall x \in X$, $\lambda_I(x) \notin \left(I_A^-(x),\ I_A^+(x)\right) \forall x \in X$ and $\lambda_F(x) \notin \left(F_A^-(x), F_A^+(x)\right) \forall x \in X$.

Definition 2.4[11] Let $\mathbb{A} = (A, \lambda)$ and $\mathbb{B} = (B, \gamma)$ be Pythagorean Neutrosophic Cubic Sets in a non empty set X where $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$,$B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle : x \in X\}$ are Pythagorean Neutrosophic Interval valued sets and $\lambda = \{\langle x, \lambda_T(x), \lambda_I(x), \lambda_F(x)\rangle : x \in X\}$, $\gamma = \{\langle x, \gamma_T(x), \gamma_I(x), \gamma_F(x)\rangle : x \in X\}$ be Pythagorean Neutrosophic Sets. Then we define the equality, $\mathbb{P}$-order and $\mathbb{R}$-order of PNCS as follows:

a) $\mathbb{A} = \mathbb{B} \Leftrightarrow A = B$ and $\lambda = \gamma$

b) $\mathbb{A} \subseteq_P \mathbb{B} \Leftrightarrow A \subseteq B$and $\lambda \leq \gamma$

c) $\mathbb{A} \subseteq_R \mathbb{B} \Leftrightarrow A \subseteq B$and $\lambda \geq \gamma$

Definition 2.5[11] For any Pythagorean Neutrosophic Cubic Sets $\mathbb{C}_i = (C_i, \lambda_i)$ in a non-empty set X where $C_i = \left\{\langle x, T_{C_i}(x), I_{C_i}(x), F_{C_i}(x)\rangle : x \in X\right\}$ and $\lambda_i = \left\{\langle x, \lambda_{T_i}(x), \lambda_{I_i}(x), \lambda_{F_i}(x)\rangle : x \in X\right\}$ for $i \in \mathbb{J}$ and $\mathbb{J}$ is any index set, we define

(a) P-union:

$$\bigcup_{i \in \mathbb{J}}^{p} \mathbb{C}_i = \left\{\langle x, \left(\cup_{j \in \mathbb{J}} C_i\right)(x), \vee_{i \in \mathbb{J}} \lambda_i(x)\rangle : x \in X\right\}$$

$$= \left\{ \left(\begin{array}{c} x, \left[\max_{i\in \mathbb{J}} T_{C_i}(x), \min_{i\in \mathbb{J}} I_{C_i}(x), \min_{i\in \mathbb{J}} F_{C_i}(x) \right], \\ \left(\max_{i\in \mathbb{J}} \lambda_{T_i}(x), \min_{i\in \mathbb{J}} \lambda_{I_i}(x), \min_{i\in \mathbb{J}} \lambda_{F_i}(x) \right) \end{array} \right) : x \in X \right\}$$

(b) P-intersection:

$$\bigcap_{\substack{P \\ i\in \mathbb{J}}} \mathbb{C}_i = \{ \langle x, (\cap_{i\in \mathbb{J}} C_i)(x), (\wedge_{i\in \mathbb{J}} \lambda_i)(x) \rangle : x \in X \}$$

$$= \left\{ \left(\begin{array}{c} x, \left[\min_{i\in \mathbb{J}} T_{C_i}(x), \max_{i\in \mathbb{J}} I_{C_i}(x), \max_{i\in \mathbb{J}} F_{C_i}(x) \right], \\ \left(\min_{i\in \mathbb{J}} \lambda_{T_i}(x), \max_{i\in \mathbb{J}} \lambda_{I_i}(x), \max_{i\in \mathbb{J}} \lambda_{F_i}(x) \right) \end{array} \right) : x \in X \right\}$$

(c) R-union:

$$\bigcup_{\substack{R \\ i\in \mathbb{J}}} \mathbb{C}_i = \{ \langle x, (\cup_{j\in \mathbb{J}} C_i)(x), \wedge_{i\in \mathbb{J}} \lambda_i(x) \rangle : x \in X \}$$

$$= \left\{ \left(\begin{array}{c} x, \left[\max_{i\in \mathbb{J}} T_{C_i}(x), \min_{i\in \mathbb{J}} I_{C_i}(x), \min_{i\in \mathbb{J}} F_{C_i}(x) \right], \\ \left(\min_{i\in \mathbb{J}} \lambda_{T_i}(x), \max_{i\in \mathbb{J}} \lambda_{I_i}(x), \max_{i\in \mathbb{J}} \lambda_{F_i}(x) \right) \end{array} \right) : x \in X \right\}$$

(d) R-intersection:

$$\bigcap_{\substack{R \\ i\in \mathbb{J}}} \mathbb{C}_i = \{ \langle x, (\cap_{i\in \mathbb{J}} C_i)(x), (\vee_{i\in \mathbb{J}} \lambda_i)(x) \rangle : x \in X \}$$

$$= \left\{ \left(\begin{array}{c} x, \left[\min_{i\in \mathbb{J}} T_{C_i}(x), \max_{i\in \mathbb{J}} I_{C_i}(x), \max_{i\in \mathbb{J}} F_{C_i}(x) \right], \\ \left(\max_{i\in \mathbb{J}} \lambda_{T_i}(x), \min_{i\in \mathbb{J}} \lambda_{I_i}(x), \min_{i\in \mathbb{J}} \lambda_{F_i}(x) \right) \end{array} \right) : x \in X \right\}$$

Definition 2.6[11] The complement of PNCS $\mathbb{A} = (A, \alpha)$ is $\mathbb{A}^c = (A^c, \alpha^c)$. $\mathbb{A}^c = \{\langle x, ([F_A^-(x), F_A^+(x)], [1 - I_A^-(x), 1 - I_A^+(x)], [T_A^-(x), T_A^+(x)]), (\lambda_F(x), 1 - \lambda_I(x), \lambda_T(x)) \rangle : x \in X\}$; $(\mathbb{A}^c)^c = \mathbb{A}$ and $\hat{0}^c = \hat{1}$; $\hat{1}^c = \hat{0}^c$ and $(\cup_P \mathbb{A}_i)^c = \cap_P \mathbb{A}_i^c$ and $(\cap_p \mathbb{A}_i)^c = \cup_P \mathbb{A}_i^c$. $(\cup_R \mathbb{A}_i)^c = \cap_R \mathbb{A}_i^c$ and $(\cap_R \mathbb{A}_i)^c = \cup_R \mathbb{A}_i^c$.

3. PYTHAGOREAN NEUTROSOPHIC CUBIC TOPOLOGY UNDER P-ORDER (P-PYTHAGOREAN NEUTROSOPHIC CUBIC TOPOLOGY)

Definition 3.1 A P-Pythagorean Neutrosophic Cubic Topology (P-PNCT) is the family F_p of Pythagorean Neutrosophic Cubic Sets in X which satisfies the following conditions:

i. $\hat{0}, \hat{1} \in F_P$

ii. Let $A_i \in F_P$, Then $\cup_P A_i \in F_P$

iii. Let $A, B \in F_P$, Then $A \cap_P B \in F_p$

Then the pair (X, F_p) is called P-Pythagorean Neutrosophic Cubic Topological Space (P-PNCTS)

Example 3.2 Let X be a non-empty set and F_P be the collection of Pythagorean Neutrosophic Cubic Sets in X. i.e., Let $X = \{a, b\}$ then $F_P = \{\hat{0}, \hat{1}, A_1, A_2\}$.

X	μ_1	λ_1
a	[0.2, 0.4], [0.3, 0.5], [0.4, 0.6]	(0.2, 0.3, 0.5)
b	[0.1, 0.4], [0.3, 0.7], [0.4, 0.8]	(0.3, 0.4, 0.7)

$\mathbb{A}_1 = (\mu_1, \lambda_1)$

X	μ_2	λ_2
a	[0.2, 0.3], [0.4, 0.8], [0.3, 0.5]	(0.2, 0.5, 0.4)
b	[0.1, 0.3], [0.3, 0.5], [0.5, 0.7]	(0.2, 0.4, 0.6)

$\mathbb{A}_2 = (\mu_2, \lambda_2)$

X	$\mu_1 \cap \mu_2$	$\lambda_1 \cap \lambda_2$
a	[0.2, 0.3], [0.4, 0.8], [0.4, 0.6]	(0.2, 0.5, 0.5)
b	[0.1, 0.3], [0.3, 0.7], [0.5, 0.8]	(0.2, 0.4, 0.7)

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$$\mathbb{A}_1 \cap \mathbb{A}_2 = (\mu_1 \cap \mu_2, \lambda_1 \cap \lambda_2)$$

X	$\mu_1 \cup \mu_2$	$\lambda_1 \cup \lambda_2$
a	$[0.2, 0.4], [0.3, 0.5], [0.3, 0.5]$	$(0.2, 0.3, 0.4)$
b	$[0.1, 0.4], [0.2, 0.5], [0.4, 0.7]$	$(0.3, 0.4, 0.6)$

$$\mathbb{A}_1 \cup \mathbb{A}_2 = (\mu_1 \cup \mu_2, \lambda_1 \cup \lambda_2)$$

$\therefore F_p$ is a P-Pythagorean Neutrosophic Cubic Topology(P-PNCT) on X, as it satisfies the necessary three axioms of topology and (X, F_P) is a P-Pythagorean Neutrosophic Cubic Topological Space(P-PNCTS)

Definition 3.3 Let X be non-empty set then F_P be the collection of all PNC-subset in X. Then we observe that F_P satisfies all the axioms of topology on X. This topology is called P-discrete PNCT and (X, F_P) is called P-discrete PNCTS.

Definition 3.4 As every PNCT on X must contain $\hat{0}, \hat{1}$ $i.e.$, $\hat{0}, \hat{1} \in F_P$, so the family $F_P = \{\hat{0}, \hat{1}\}$ forms a PNCT on X. The Topology is called P-indiscrete PNCT and (X, F_P) is called a P-indiscrete PNCTS.

Example 3.5 Let X be a non-empty set and $F_P = \{\hat{0}, \hat{1}\}$ be the collection of Pythagorean Neutrosophic Cubic Sets, Then F_P is obviously a P-PNCT on X which is the smallest P-PNCT on X. This P-PNCT is P-indiscrete PNCT.

Theorem 3.6 Let X be a non-empty set and $F_P = \{A_X\}$ be the collection of all possible PNC subset of X. Then F_P is P-PNCT on X and is called as P-discrete PNCT.

Proof

Since $\hat{0}, \hat{1}$ are PNC-subsets of X. Then $\hat{0}, \hat{1} \in F_P$. Let $\{A_i \backslash i \in \mathbb{N}\}$ be the family of PNC subsets of X. Then $\bigcup_{i \in N}^{p} A_i$ is also a PNC-subset of X. Then $\bigcap_{i \in \mathbb{N}}^{P} A_i$ is also a PNC-subset of X. Hence $\bigcup_{i \in N}^{p} A_i \in F_P$ and $\bigcap_{i \in \mathbb{N}}^{P} A_i \in F_P \Rightarrow F_P$ is P-PNCT on X.

Definition 3.7 The members of P-Pythagorean Neutrosophic Cubic Topology (F_P) is called P-Pythagorean Neutrosophic Cubic Open Sets in (X, F_P).

Example 3.8 Let $X = \{a, b\}$ be a non-empty set and $F_P = \left\{ \begin{matrix} \hat{0}, \hat{1}, \langle a, [0.2, 0.4], [0.3, 0.5], [0.4, 0.6], (0.2, 0.3, 0.5) \rangle, \\ \langle b, [0.1, 0.4], [0.3, 0.7], [0.4, 0.8], (0.3, 0.4, 0.7) \rangle \end{matrix} \right\}$ be P-Pythagorean Neutrosophic Cubic Topology on X. Then

$\hat{0}, \hat{1}, \langle a, [0.2, 0.4], [0.3, 0.5], [0.4, 0.6], (0.2, 0.3, 0.5) \rangle, \langle b, [0.1, 0.4], [0.3, 0.7], [0.4, 0.8], (0.3, 0.4, 0.7) \rangle$ are P-Pythagorean Neutrosophic Cubic Open sets in (X, F_P).

Proposition 3.9 If (X, F_P) is any P-Pythagorean Neutrosophic Cubic Topological Space, then

i. $\hat{0}$ and $\hat{1}$ are P-Pythagorean Neutrosophic Cubic Open Sets.

ii. The P-union of any (finite or infinite) number of P-Pythagorean Neutrosophic Cubic Open sets is a P-Pythagorean Neutrosophic Open Set.

iii. The P-intersection of any finite P-Pythagorean Neutrosophic Cubic open sets is a P-Pythagorean Neutrosophic Cubic Open set.

Proof

i. From the definition 3.1, P-Pythagorean Neutrosophic Cubic Topology, $\hat{0}, \hat{1} \in F_P$, Hence $\hat{0}$ and $\hat{1}$ are P-Pythagorean Neutrosophic Cubic Open Sets.

ii. Let $A_i \backslash i \in \mathbb{N}$ be P-Pythagorean Neutrosophic Cubic Open sets, Then $A_i \in F_P$. By definition 3.1, $\bigcup_{i \in \mathbb{N}}^{P} A_i \in F_P$. Hence $\bigcup_{i \in \mathbb{N}}^{P} A_i$ is P-Pythagorean Neutrosophic Cubic Open Set.

iii. Let $A_1, A_2, \dots A_n$ be P-Pythagorean Neutrosophic Cubic Open Sets, Then by definition 3.1, $\bigcap_{i \in \mathbb{N}}^{P} A_i \in F_P$. Hence $\bigcap_{i \in \mathbb{N}}^{P} A_i$ is P-Pythagorean Neutrosophic Cubic Open Set.

Definition 3.10 The complement of P-Pythagorean Neutrosophic Cubic open sets is called P-Pythagorean Neutrosophic Cubic Closed sets in (X, F_P).

Example 3.11 Let X be a non-empty set. Let $F_P = \begin{Bmatrix} \hat{0}, \hat{1}, \langle a, [0.2, 0.4], [0.3, 0.5], [0.4, 0.6], (0.2, 0.3, 0.5) \rangle, \\ \langle b, [0.1, 0.4], [0.3, 0.7], [0.4, 0.8], (0.3, 0.4, 0.7) \rangle \end{Bmatrix}$ be P-PNCT on X. Then $\hat{0}, \hat{1}, \langle a, [0.4, 0.6], [0.5, 0.7], [0.2, 0.4], (0.5, 0.7, 0.2) \rangle, \langle b, [0.4, 0.8], [0.3, 0.7], [0.1, 0.4], (0.7, 0.6, 0.3) \rangle$ are P-PNCCS(P-Pythagorean Neutrosophic Cubic Closed Sets).

Proposition 3.12 If X, F_P is any P-Pythagorean Neutrosophic Cubic Topological Space, then

i. $\hat{0}$ and $\hat{1}$ are P-Pythagorean Neutrosophic Cubic Closed Sets.

ii. The P-intersection of any (finite or infinite) number of P-Pythagorean Neutrosophic Cubic closed sets is a P-Pythagorean Neutrosophic Cubic Closed set.

iii. The P-union of any number of P-Pythagorean Neutrosophic Cubic Closed sets is a P-Pythagorean Neutrosophic Closed Set.

Proof

i. From the definition 3.1, P-Pythagorean Neutrosophic Cubic Topology, $\hat{0}, \hat{1} \in F_P$. Since the complement of $\hat{0}$ is $\hat{1}$ and $\hat{1}$ is $\hat{0}$, This implies $\hat{0}$ and $\hat{1}$ are P-Pythagorean Neutrosophic Cubic Closed Sets.

ii. Let $A_i \backslash i \in \mathbb{N}$ be P-Pythagorean Neutrosophic Cubic Closed sets, Then $A_i^C \in F_P$. By definition 3.1, $\bigcup_{i\in\mathbb{N}}^{P} A_i^C \in F_P$. This implies $\bigcup_{i\in\mathbb{N}}^{P} A_i^C$ is P-Pythagorean Neutrosophic Cubic Open Set but $\bigcup_{i\in\mathbb{N}}^{P} A_i^C = \left(\bigcap_{i\in\mathbb{N}}^{P} A_i \right)^C$. Hence $\bigcap_{i\in\mathbb{N}}^{P} A_i$ is P-Pythagorean Neutrosophic Cubic Closed Set.

iii. Let $A_1, A_2, \dots A_n$ be P-Pythagorean Neutrosophic Cubic Closed Sets, implies that $A_1^C, A_2^C, \dots A_n^C$ are P-Pythagorean Neutrosophic Cubic Open Sets, i.e., $A_1^C, A_2^C, \dots A_n^C \in F_P$ Then by definition 3.1, $\bigcap_{i\in\mathbb{N}}^{P} {A_i}^C \in F_P$. This implies $\bigcap_{i\in\mathbb{N}}^{P} {A_i}^C$ is P-Pythagorean Neutrosophic Cubic Open Set but $\bigcap_{i\in\mathbb{N}}^{P} {A_i}^C = \left(\bigcup_{i\in\mathbb{N}}^{P} A_i \right)^C$. Hence $\bigcup_{i\in\mathbb{N}}^{P} A_i$ is P-Pythagorean Neutrosophic Cubic Closed Set.

Definition 3.13 PNCS which are both P-PNCO and P-PNCC are called P-Pythagorean Neutrosophic Clopen Sets in (X, F_P).

Example 3.14 Let X be a non-empty set and $F_P = \begin{Bmatrix} \hat{0}, \hat{1}, \langle [0.2, 0.4][0.3,0.5][0.4, 0.6], (0.2, 0.3, 0.5) \rangle \\ \langle [0.3, 0.7][0.4, 0.6][0.3, 0.7], (0.6, 0.5, 0.6) \rangle \end{Bmatrix}$ be a P-PNCT on X, Then $\hat{0}, \hat{1}$ and $\langle [0.3, 0.7][0.4, 0.6][0.3, 0.7], (0.6, 0.5, 0.6) \rangle$ are P-Pythagorean Neutrosophic Clopen Sets in (X, F_P).

Proposition 3.15

i. In every P-Pythagorean Neutrosophic Cubic Topological Space (X, F_P), $\hat{0}$ and $\hat{1}$ are P-Pythaogrean Neutrosophic Cubic clopen sets.

ii. In a discrete P-Pythagorean Neutrosophic Cubic Topological Space all the Pythagorean Neutrosophic cubic subset of X are P-Pythagorean Neutrosophic Cubic clopen sets.

iii. In an indiscrete P-Pythagorean Neutrosophic Cubic Topological Space the only P-Pythagorean Neutrosophic Cubic clopen sets are $\hat{0}$ and $\hat{1}$.

Proof : The proof of the above are trivial.

Definition 3.16 A P-Pythagorean Neutrosophic Cubic topology F_{P_2} is P-Pythagorean Neutrosophic Cubic coarser than F_{P_1} if and only if $F_{P_2} \subset_P F_{P_1}$.

Example 3.17 Let X be a non-empty set. Now, $F_{P_1} = \{\hat{0}, \hat{1}, \langle [0.2, 0.4][0.3,0.5][0.4, 0.6], (0.2, 0.3, 0.5) \rangle\}$ and $F_{P_2} = \{\hat{0}, \hat{1}\}$ be two topologies on X, Then $F_{P_2} \subset_P F_{P_1}$. Hence F_{P_2} is P-Pythagorean Neutrosophic Cubic Coarser that F_{P_1}.

Definition 3.18 A Pythagorean Neutrosophic Cubic Set U_P in (X, F_P) is a P-Pythagorean Neutrosophic Cubic neighborhood set A if and only if there exists an P-Pythagorean Neutrosophic Cubic Open set O_P in (X, F_P) such that $A \subset_P O_P \subset_P U_P$.

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Definition 3.19 The family of all P-Pythagorean Neutrosophic Cubic neighborhoods U_{P_i} of a Pythagorean Neutrosophic Cubic Set A in X is called P-Pythagorean Neutrosophic Cubic neighborhood system. i.e., $\exists O_{P_i}$ such that $A \subset_P O_{P_i} \subset_P U_{P_i}$ where $i \in \mathbb{N}$. In other words U_{P_i} is P-Pythagorean Neutrosophic Cubic Neighborhood of A if $A \subset_P U_{P_i}$ because $\hat{0}$ is a subset of all P-Pythagorean Neutrosophic Cubic Sets.

Definition 3.20 A collection F_P of Internal Pythagorean Neutrosophic Cubic Sets (IPNCS) and satisfies the following conditions,

i. $\hat{0}, \hat{1} \in F_P$.

ii. Let $A_i \in F_P$, then $\bigcup_{\substack{P \\ i\in\mathbb{N}}} A_i \in F_P$

iii. Let $A, B \in F_P$, then $A \cap_P B \in F_p$ is called Internal P-Pythagorean Neutrosophic Cubic Topology. (IPPNCT)

Example 3.21 Let $F_P = \{\mathbb{A}_i = \langle A_i, \lambda_i \rangle \backslash i \in \mathbb{N}\}$ be family of Internal Pythagorean Neutrosophic Cubic Sets in X. Then F_P is a P-Pythagorean Neutrosophic Cubic Topology on X and is called as Internal P-Pythagorean Neutrosophic Cubic topology (IPPNCT)

Proof: Since $\hat{0} = \langle 0, 0 \rangle$ and $\hat{1} = \langle 1,1 \rangle$. Now, $\hat{0} \cup_P \mathbb{A}_i = \mathbb{A}_i \in F_P$ and $\hat{1} \cup_P \mathbb{A}_i = \hat{1} \in F_P$. Also $\hat{0} \cap_P \mathbb{A}_i = \hat{0} \in F_P$ and $\hat{1} \cap_P \mathbb{A}_i = \mathbb{A}_i \in F_P$. Since $\mathbb{A}_i$ is an IPNCS in X so we have $A_i^-(x) \leq \lambda_i(x) \leq A_i^+(x)$ for $i \in \mathbb{N}$. This implies, $(\bigcup_{i\in\mathbb{N}} A_i^-)(x) \leq (\bigvee_{i\in\mathbb{N}} \lambda_i)(x) \leq (\bigcup_{i\in\mathbb{N}} A_i^+)(x)$ then for $i \in \mathbb{N}$ implies that $\bigcup_{\substack{P \\ i\in\mathbb{N}}} \mathbb{A}_i$ is an Internal P-Pythagorean Neutrosophic Cubic Set in X. And $(\bigcap_{i=1}^n A_i^-)(x) \leq \left(\bigwedge_{i=1}^n \lambda_i(x)\right) \leq (\bigcap_{i=1}^n A_i^+)(x)$ then for $i \in \mathbb{N}$ implies that $\bigcap_{\substack{i=1 \\ P}}^n \mathbb{A}_i$ is an Internal P-Pythagorean Neutrosophic Cubic Set in X. Hence F_P is P- Internal Pythagorean Neutrosophic Cubic Topology on X.

Remark 3.22 The collection of EPNCS need not to be P-PNCT.

Example 3.23 Let F_P be the collection of EPNCS. Let $\mathbb{A}$ and $\mathbb{B}$ are two EPNCS in F_P where $A = \{[0.3, 0.5], [0.2, 0.4], [0.1,0.2]\}$, $\lambda_A = (0.8, 0.1, 0.3)$ and $B = \{[0.7, 0.8], [0.1, 0.2], [0.2, .4]\}$, $\lambda_B = (0.4, 0.3, 0.1)$. $\mathbb{A} \cup_P \mathbb{B} = \{[0.7, 0.8][[0.1, 0.2], [0.1, 0.2], (0.8, 0.1, 0.1)\}$ and $\mathbb{A} \cap_P \mathbb{B} = \{[0.3, 0.5], [0.2, 0.4], [0.2, 0.4], (0.4, 0.3, 0.3)\}$ which are not EPNCS because $\lambda_A \in A$ and $\lambda_B \in B$. $\therefore F_P$ is not a P-Pythagorean Neutrosophic Cubic Topology in X.

Definition 3.24 Let F_P be the collection of IPNCS and EPNCS. Then under P-order F_P is a P-PNCT.

Example 3.25 Let $F_P = \{C^X\}$, where C^X denotes all possible PNC-subsets of X. Since $\hat{0}, \hat{1} \in F_P$ and $\bigcup_{\substack{P \\ i\in\mathbb{N}}} C_i \in F_P$ is either IPNCS or EPNCS in X. So $\bigcup_{\substack{P \\ i\in\mathbb{N}}} C_i \in F_P$. Now $\bigcap_{i=1}^n C_i$ is either IPNCS or EPNCS in X. So $\bigcap_{i=1}^n C_i \in F_P$. Then F_P is P-PNCT on X.

Theorem 3.26 Let $\mathbb{C} = \langle C, \lambda \rangle \in F_P$ be Internal P-PNCOS, Then $\mathbb{C}^c$ is also Internal P-PNCCS.

Proof Let $\mathbb{C} = \langle C, \lambda \rangle$ be Internal P-PNCOS in X. Then $C^-(x) \leq \lambda(x) \leq C^+(x) \forall x \in X$ this implies $1 - C^+(x) \leq 1 - \lambda(x) \leq 1 - A^-(x) \forall x \in X$ which is an Internal P-PNCS implies that $\mathbb{C}^c$ is Internal P-PNCCS.

Theorem 3.27 Let $\mathbb{C} = \langle C, \lambda \rangle \in F_P$ be External P-PNCOS, Then $\mathbb{C}^c$ is also External P-PNCCS.

Proof Let $\mathbb{C} = \langle C, \lambda \rangle$ be External P-PNCOS in X. Then $\lambda(x) \notin \left(C^-(x), C^+(x)\right) \forall x \in X$ this implies $1 - \lambda(x) \notin \left(1 - A^+(x), 1 - A^-(x)\right) \forall x \in X$ which is an External P-PNCS implies that $\mathbb{C}^c$ is External P-PNCCS.

4. PYTHAGOREAN NEUTROSOPHIC CUBIC TOPOLOGY UNDER R-ORDER (R-PYTHAGOREAN NEUTROSOPHIC CUBIC TOPOLOGY)

Definition 4.1 A R-Pythagorean Neutrosophic Cubic Topology (R-PNCT) is the family F_R of Pythagorean Neutrosophic Cubic Sets in X which satisfies the following conditions:

i. $\hat{0}, \hat{1} \in F_R$

ii. Let $A_i \in F_R$, Then $\bigcup_R A_i \in F_R$

iii. Let $A, B \in F_R$, Then $A \bigcap_R B \in F_R$

Then the pair (X, F_R) is called R-Pythagorean Neutrosophic Cubic Topological Space (R-PNCTS)

Example 4.2 Let X be a non-empty set and F_R be the collection of Pythagorean Neutrosophic Cubic Sets in X. i.e., Let $X = \{a\}$ then $F_R = \left\{\hat{0}, \hat{1}, \langle [0.3, 0.5], [0.2, 0.4][0.1, 0.2], (0.8, 0.1, 0.3) \rangle, \langle [0.7, 0.8], [0.1, 0.2], [0.2, 0.4], (0.4, 0.3, 0.1) \rangle\right\}$.

Then F_P is R-PNCT on X.

Definition 4.3 Let X be non-empty set then F_R be the collection of all PNC-subset in X. Then we observe that F_R satisfies all the axioms of topology on X. This topology is called R-discrete PNCT and (X, F_R) is called P-discrete PNCTS.

Definition 4.4 As every PNCT on X must contain $\hat{0}, \hat{1}$ $i.e.$, $\hat{0}, \hat{1} \in F_R$, so the family $F_R = \{\hat{0}, \hat{1}\}$ forms a PNCT on X. The Topology is called R-indiscrete PNCT and (X, F_P) is called a R-indiscrete PNCTS.

Definition 4.5 The members of P-Pythagorean Neutrosophic Cubic Topology (F_R) is called R-Pythagorean Neutrosophic Cubic Open Sets in (X, F_R).

Example 4.6 Let $X = \{a\}$ be a non-empty set and $F_R = \{\hat{0}, \hat{1}, \langle [0.3, 0.5], [0.2, 0.4][0.1, 0.2], (0.8, 0.1, 0.3) \rangle, \langle [0.7, 0.8], [0.1, 0.2], [0.2, 0.4], (0.4, 0.3, 0.1) \rangle\}$. be R-Pythagorean Neutrosophic Cubic Topology on X. Then $\hat{0}, \hat{1}, \langle [0.3, 0.5], [0.2, 0.4][0.1, 0.2], (0.8, 0.1, 0.3) \rangle, \langle [0.7, 0.8], [0.1, 0.2], [0.2, 0.4], (0.4, 0.3, 0.1) \rangle$ are R-Pythagorean Neutrosophic Cubic Open sets in (X, F_R).

Proposition 4.7 If (X, F_R) is any R-Pythagorean Neutrosophic Cubic Topological Space, then

i. $\hat{0}$ and $\hat{1}$ are R-Pythagorean Neutrosophic Cubic Open Sets.

ii. The R-union of any finite number of R-Pythagorean Neutrosophic Cubic Open sets is a R-Pythagorean Neutrosophic Open Set.

iii. The R-intersection of any finite R-Pythagorean Neutrosophic Cubic open sets is a R-Pythagorean Neutrosophic Cubic Open set.

Proof

i. From the definition 4.1, R-Pythagorean Neutrosophic Cubic Topology, $\hat{0}, \hat{1} \in F_R$, Hence $\hat{0}$ and $\hat{1}$ are R-Pythagorean Neutrosophic Cubic Open Sets.

ii. Let $A_i \backslash i \in \mathbb{N}$ be R-Pythagorean Neutrosophic Cubic Open sets,Then $A_i \in F_R$. By definition 4.1, $\bigcup_{i \in \mathbb{N}}^{R} A_i \in F_R$. Hence $\bigcup_{i \in \mathbb{N}}^{R} A_i$ is R-Pythagorean Neutrosophic Cubic Open Set.

iii. Let $A_1, A_2, \dots A_n$ be R-Pythagorean Neutrosophic Cubic Open Sets. Then $A_1, A_2, \dots A_n \in F_R$. Then by definition 4.1, $\bigcap_{i \in \mathbb{N}}^{R} A_i \in F_R$. Hence $\bigcap_{i \in \mathbb{N}}^{R} A_i$ is R-Pythagorean Neutrosophic Cubic Open Set.

Definition 4.8 The complement of R-Pythagorean Neutrosophic Cubic open sets is called R-Pythagorean Neutrosophic Cubic Closed sets in (X, F_R).

Example 4.9 Let $X = \{a\}$ be a non-empty set and $F_R = \{\hat{0}, \hat{1}, \langle [0.3, 0.5], [0.4, 0.5][0.2, 0.3], (0.7, 0.2, 0.15) \rangle\}$ be R-PNCT on X. Then $\hat{1}, \hat{0}, \langle [0.2, 0.3], [0.5, 0.6], [0.3, 0.5], (0.15, 0.8, 0.7) \rangle$ are R-Pythagorean Neutrosophic Cubic Closed Sets in (X, F_R).

Proposition 4.10 If X, F_R is any R-Pythagorean Neutrosophic Cubic Topological Space, then

i. $\hat{0}$ and $\hat{1}$ are R-Pythagorean Neutrosophic Cubic Closed Sets.

ii. The R-intersection of any (finite or infinite) number of R-Pythagorean Neutrosophic Cubic closed sets is a R-Pythagorean Neutrosophic Cubic Closed set.

iii. The R-union of any number of R-Pythagorean Neutrosophic Cubic Closed sets is a R-Pythagorean Neutrosophic Closed Set.

Proof

i. From the definition 4.1, R-Pythagorean Neutrosophic Cubic Topology, $\hat{0}, \hat{1} \in F_{\mathrm{R}}$. Since the complement of $\hat{0}$ is $\hat{1}$ and $\hat{1}$ is $\hat{0}$, This implies $\hat{0}$ and $\hat{1}$ are R-Pythagorean Neutrosophic Cubic Closed Sets.

ii. Let $A_i \backslash i \in \mathbb{N}$ be R -Pythagorean Neutrosophic Cubic Closed sets, Then $A_i^C \in F_{\mathrm{R}}$. By definition 4.1, $\bigcup_{i \in \mathbb{N}}^{\mathrm{R}} A_i^C \in F_{\mathrm{R}}$. This implies $\bigcup_{i \in \mathbb{N}}^{\mathrm{R}} A_i^C$ is R -Pythagorean Neutrosophic Cubic

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Open Set but $\bigcup_{i\in\mathbb{N}}^{\mathrm{R}} A_i^C = \left(\bigcap_{i\in\mathbb{N}}^{\mathrm{R}} A_i\right)^C$. Hence $\bigcap_{i\in\mathbb{N}}^{\mathrm{R}} A_i$ is R -Pythagorean Neutrosophic Cubic Closed Set.

iii. Let $A_1, A_2, \dots A_n$ be R -Pythagorean Neutrosophic Cubic Closed Sets, implies that $A_1^C, A_2^C, \dots A_n^C$ are R -Pythagorean Neutrosophic Cubic Open Sets, i.e., $A_1^C, A_2^C, \dots A_n^C \in F_{\mathrm{R}}$ Then by definition 4.1, $\bigcap_{i\in\mathbb{N}}^{\mathrm{R}} {A_i}^C \in F_{\mathrm{R}}$. This implies $\bigcap_{i\in\mathbb{N}}^{\mathrm{R}} {A_i}^C$ is R -Pythagorean Neutrosophic Cubic Open Set but $\bigcap_{i\in\mathbb{N}}^{\mathrm{R}} {A_i}^C = \left(\bigcup_{i\in\mathbb{N}}^{\mathrm{R}} A_i\right)^C$. Hence $\bigcup_{i\in\mathbb{N}}^{\mathrm{R}} A_i$ is R -Pythagorean Neutrosophic Cubic Closed Set.

Definition 4.11 PNCS which are both R -PNCO and R -PNCC are called R -Pythagorean Neutrosophic Clopen Sets in (X, F_{R}).

Example 4.12 Let X be a non-empty set and $F_R = \left\{\begin{matrix}\hat{0}, \hat{1}, \langle[0.3, 0.7][0.4, 0.6][0.3, 0.7], (0.6, 0.5, 0.6)\rangle \\ \langle[0.2, 0.4][0.3,0.5][0.4, 0.6], (0.2, 0.3, 0.5)\rangle\end{matrix}\right\}$ be a R-PNCT on X, Then $\hat{0}, \hat{1}$ and $\langle[0.3, 0.7][0.4, 0.6][0.3, 0.7], (0.6, 0.5, 0.6)\rangle$ are R-Pythagorean Neutrosophic Clopen Sets in (X, F_R).

Proposition 4.13

i. In every R-Pythagorean Neutrosophic Cubic Topological Space (X, F_R), $\hat{0}$ and $\hat{1}$ are R-Pythagorean Neutrosophic Cubic clopen sets.

ii. In a discrete R-Pythagorean Neutrosophic Cubic Topological Space all the Pythagorean Neutrosophic cubic subset of X are R-Pythagorean Neutrosophic Cubic clopen sets.

iii. In an indiscrete R-Pythagorean Neutrosophic Cubic Topological Space the only R-Pythagorean Neutrosophic Cubic clopen sets are $\hat{0}$ and $\hat{1}$.

Proof : The proof of the above are trivial.

Definition 4.14 A R-Pythagorean Neutrosophic Cubic topology F_{R_2} is R-Pythagorean Neutrosophic Cubic coarser than F_{R_1} if and only if $F_{R_2} \subset_P F_{R_1}$.

Example 4.15 Let X be a non-empty set. Now, $F_{R_1} = \{\hat{0}, \hat{1}, \langle[0.1, 0.4][0.3,0.5][0.4, 0.6], (0.2, 0.5, 0.5)\rangle\langle[0.2, 0.3], [0.4, 0.5], [0.5, 0.6], (0.2, 03, 0.5)\rangle\}$ and $F_{R_2} = \{\hat{0}, \hat{1}\}$ be two topologies on X, Then $F_{R_2} \subset_R F_{R_1}$. Hence F_{R_2} is R-Pythagorean Neutrosophic Cubic Coarser that F_{R_1}.

Definition 4.16 A Pythagorean Neutrosophic Cubic Set U_R in (X, F_R) is a R-Pythagorean Neutrosophic Cubic Neighborhood set A if and only if there exists an R-Pythagorean Neutrosophic Cubic Open set O_R in (X, F_R) such that $A \subset_R O_R \subset_R U_R$.

Definition 4.17 The family of all R-Pythagorean Neutrosophic Cubic neighborhoods U_{R_i} of a Pythagorean Neutrosophic Cubic Set A in X is called R-Pythagorean Neutrosophic Cubic neighborhood system. i.e., $\exists O_{R_i}$ such that $A \subset_R O_{R_i} \subset_R U_{R_i}$ where $i \in \mathbb{N}$.

Remark 4.18 The collection of EPNCS need not to be R-PNCT.

Example 4.19 Let F_R be the collection of EPNCS. Let $\mathbb{A}$ and $\mathbb{B}$ are two EPNCS in F_P where $\mathbb{A} = \{[0.3, 0.5], [0.4, 0.5], [0.2,0.7], (0.95, 0.2, 0.3)\}$ and $\mathbb{B} = \{[0.7, 0.9], [0.2, 0.6], [0.4, 0.6], (0.7, 0.19, 0.15)\}$

$\mathbb{A} \cap_R \mathbb{B} = \{[0.3, 0.5], [0.4, 0.6], [0.4, 0.7], (0.95, 0.19, 0.15)\}$ which is EPNCS.

$\mathbb{A} \cup_R \mathbb{B} = \{[0.7, 0.9][[0.2, 0.5], [0.2, 0.6], (0.7, 0.2, 0.3)\}$ which are IPNCS because $0.7 \in [0.7, 0.9]$, $0.2 \in [0.2, 0.5]$ and $0.3 \in [0.2, 0.6]$. $\therefore F_R$ is not a R-Pythagorean Neutrosophic Cubic Topology in X.

Remark 4.20 The collection of IPNCS need not to be R-PNCT.

Example 4.21 Let F_R be the collection of EPNCS. Let $\mathbb{A}$ and $\mathbb{B}$ are two EPNCS in F_P where $\mathbb{A} = \{[0.1, 0.3], [0.4, 0.6], [0.5,0.8], (0.15, 0.5, 0.62)\}$ and $\mathbb{B} = \{[0.2, 0.3], [0.6, 0.8], [0.3, 0.5], (0.3, 0.65, 0.35)\}$

$\mathbb{A} \cap_R \mathbb{B} = \{[0.1, 0.3], [0.6, 0.8], [0.5, 0.8], (0.3, 0.5, 0.35)\}$ which is EPNCS.

$\mathbb{A} \cup_R \mathbb{B} = \{[0.2, 0.3][[0.4, 0.6], [0.3, 0.5], (0.15, 0.65, 0.62)\}$ which is EPNCS because $\lambda_A \notin A(x); \lambda_B \notin B(x)$. Hence $\mathbb{A} \cap_R \mathbb{B} \notin F_R$ and $\mathbb{A} \cup_R \mathbb{B} \notin F_R$.

$\therefore F_R$ is not a R-Pythagorean Neutrosophic Cubic Topology in X.

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