Wave Optics

Wave front

The wave front at any instant is defined as the locus of all the particles of the medium which are in the same state of vibration.

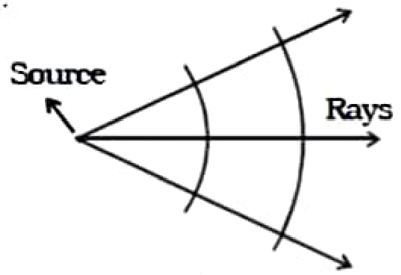
Or

Aan imaginary surface passing through particles oscillating with same phase is known as wavefront

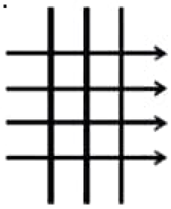
A point source of light at a finite distance in an isotropic medium emits a spherical wave front (Fig a).

A point source of light in an isotropic medium at infinite distance will give rise to plane wavefront (Fig. b).

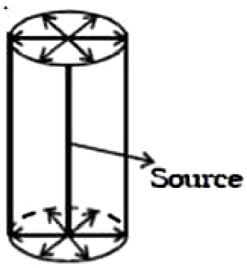
A linear source of light such as a slit illuminated by a lamp, will give rise to cylindrical wavefront (Fig c).



(a)



(b)



(c)

HUYGENS PRINCIPLE

Huygen's principle states that,

(i) every point on a given wave front may be considered as a source of secondary wavelets which spread out with the speed of light in that medium and

(ii) the new wavefront is the forward envelope of the secondary wavelets at that instant

Huygen's construction for a spherical and plane wavefront:

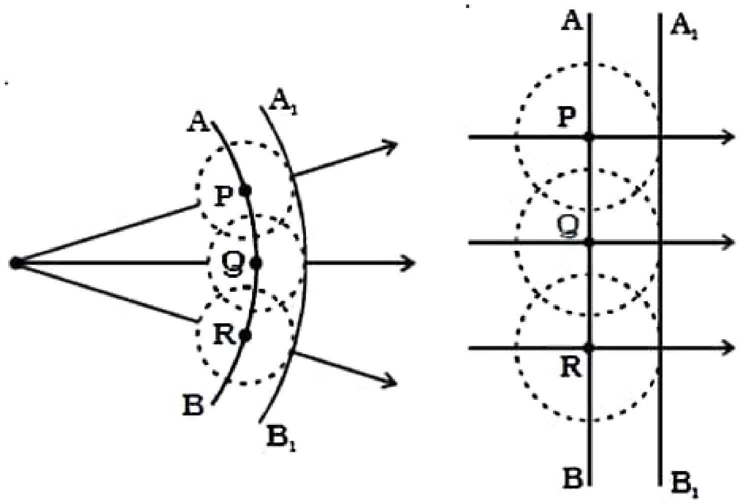
Huygen's construction for a spherical and plane wavefront is shown in Fig.a.

Let AB represent a given wavefront at a time $t = 0$. According to Huygen's principle, every point on AB acts as a source of secondary wavelets which travel with the speed of light c.

To find the position of the wave front after a time t, circles are drawn with points P, Q, R ... etc as centres on AB and radii equal to ct.

These are the traces of secondary wavelets. The arc A_1B_1 drawn as a forward envelope of the small circles is the new wavefront at that instant.

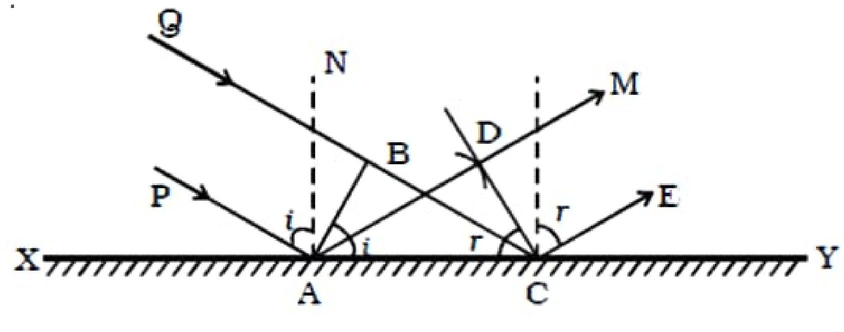
If the source of light is at a large distance, we obtain a plane wave front A_1 B_1 as shown in Fig b.



(a) (b)

Reflection of a plane wave front at a plane surface

Let XY be a plane reflecting surface and AB be a plane wavefront incident on the surface at A. PA and QBC are perpendiculars drawn to
AB at A and B respectively. Hence they represent incident rays. AN is the normal drawn to the surface.
The wave front and the surface are perpendicular to the plane of the paper (Fig.).
According to Huygen's principle each point on the wavefront acts as the source of secondary wavelet.
By the time, the secondary wavelets from B travel a distance BC, the secondary wavelets from A on the reflecting surface would travel the same distance BC after reflection.
Taking A as centre and BC as radius an arc is drawn.
From C a tangent CD is drawn to this arc. This tangent CD not only envelopes the wavelets from C and A but also the wavelets from all the points between C and A.



Therefore CD is the reflected plane wavefront and AD is the reflected ray.

Laws of reflection

(i) The incident wavefront AB, the reflected wavefront CD and the reflecting surface XY all lie in the same plane.

(ii) Angle of incidence i = $\angle$ PAN = 90^0 – $\angle$ NAB = $\angle$BAC

Angle of reflection r = $\angle$ NAD = 90^0 – $\angle$ DAC = $\angle$DCA

$\angle B = \angle D = 90^0$
BC = AD and AC is common
$\therefore$ The two triangles are congruent
$\angle$ BAC = $\angle$DCA
i.e. i = r
Thus the angle of incidence is equal to angle of reflection.

Refraction of a plane wavefront at a plane surface

Let XY be a plane refracting surface separating two media 1 and 2 of refractive indices μ_1 and μ_2 (Fig). The velocities of light in these two media are respectively v_1 and v_2.
Consider a plane wave front AB incident on the refracting surface at A. PA and QBC are perpendiculars drawn to AB at A and B respectively. Hence they represent incident rays. NAN_1 is the normal drawn to the surface. The wave front and the surface are perpendicular to the plane of the paper.
According to Huygen's principle each point on the wave front act as the source of secondary wavelet.
By the time, the secondary wavelets from B, reaches C, the secondary wavelets from the point A would travel a distance AD = v_2t, where t is the time taken by the wavelets to travel the distance BC.
$\therefore BC = C_1t$ and $AD = C_2t$
Taking A as centre and C_2t as radius an arc is drawn in the second medium. From C a tangent CD is drawn to this arc.

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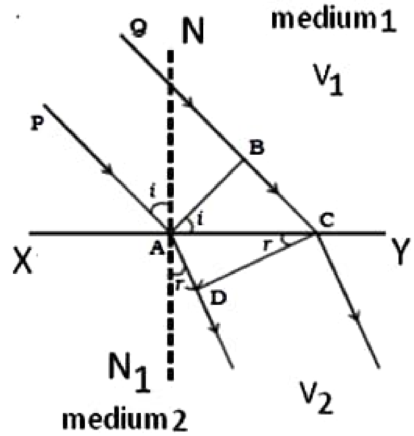
Therefore CD is the refracted plane wavefront and AD is the refracted ray



Laws of refraction

(i) The incident wave front AB, the refracted wave front CD and the refracting surface XY all lie in the same plane.
(ii) From figure for Δ ABC and Δ ACD

$$\frac{sini}{sinr} = \frac{BC/_{AC}}{AD/_{AC}} = \frac{BC}{AD} = \frac{v_1t}{v_2t} = \frac{v_1}{v_2} = n_{21}$$

Constant n_{21} in above equation is known as refractive index of medium 2 with respect to medium also represented as $_1\mu_2$

This is Snell's law of refraction

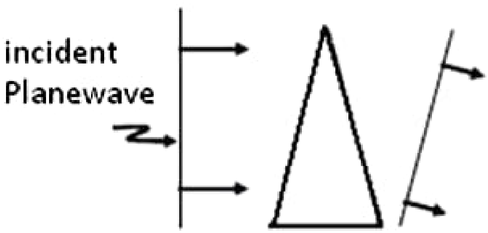
Further, if λ_1 and λ_2 denote the wavelengths of light in medium 1 and medium 2, respectively and if the distance BC is equal to λ_1 then the distance AE will be equal to λ_2 (because if the crest from B has reached C in time τ, then the crest from A should have also reached *E* in time τ); thus

$$\frac{\lambda_1}{\lambda_2} = \frac{BC}{AE} = \frac{v_1}{v_2}$$

The above equation implies that when a wave gets refracted into a denser medium ($v_1 > v_2$) the wavelength and the speed of propagation decrease but the *frequency* f (= v/λ) *remains the same.*

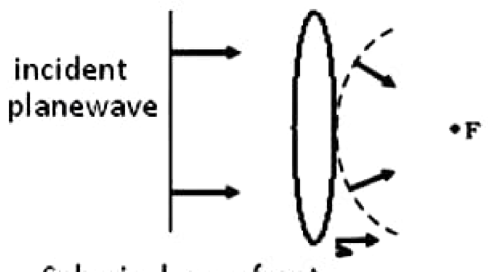
Refraction of a plane wave by a thin prism

we consider a plane wave passing through a thin prism. Clearly, since the speed of light waves is less in glass, the lower portion of the incoming wavefront (which travels through the greatest thickness of glass) will get delayed resulting in a tilt in the emerging wavefront as shown in the figure.



(b) a convex lens.

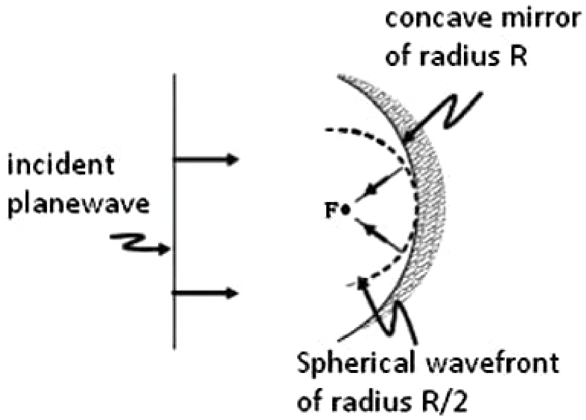
We consider a plane wave incident on a thin convex lens; the central part of the incident plane wave traverses the thickest portion of the lens and is delayed the most. The emerging wavefront has a depression at the centre and therefore the wavefront becomes spherical and converges to the point F which is known as the *focus*.



Spherical wavefront
of radius f

(c) Reflection of a plane wave by a concave mirror

a plane wave is incident on a concave mirror and on reflection we have a spherical wave converging to the focal point F.

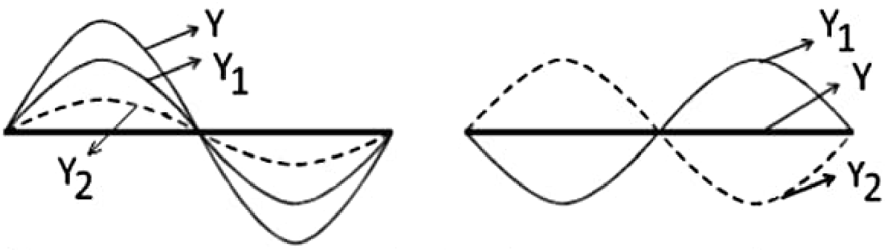


Coherent and incoherent sources

Two sources are said to be coherent if they emit light waves of the same wave length and start with same phase or have a constant phase difference.
Two independent monochromatic sources, emit waves of same wave length. But the waves are not in phase. So they are not coherent.
This is because, atoms cannot emit light waves in same phase and these sources are said to be incoherent sources.

Superposition principle

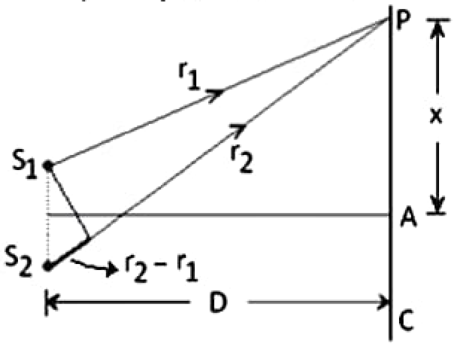
When two or more waves simultaneously pass through the same medium, each wave acts on every particle of the medium, as if the other waves are not present. The resultant displacement of any particle is the vector addition of the displacements due to the individual waves. This is known as principle of superposition. If $\mathbf{Y_1}$ and $\mathbf{Y_2}$ represent the individual displacement then the resultant displacement is given by $\mathbf{Y = Y_1 + Y_2}$



Thus, superposition principle describes a situation when more than one waves superpose (i.e. interfere) at a point.
" The effect produced by superposition of two or more wave is called interference".

Interference due to two waves

Suppose two harmonic waves having initial phase φ_1 and φ_2 are emitted from two point like sources S_1 and S_2 respectively. They superimpose simultaneously (i.e. at the same time t) at a point P as shown in figure.



Visible perception of light is produced only by electric field, and therefore, in the present case we write light waves produced by source S_1 and S_2 in terms of electric fields (e) only. Due to S_1 source,

$$\vec{e_1} = \vec{E_1} sin(\omega_1 t - k_1 r_1 + \varphi_1)$$

And due to source S_2

$$\vec{e_2} = \vec{E_2} sin(\omega_2 t - k_2 r_2 + \varphi_2)$$

Here E_1 and E_2 represent amplitude of electric fields, ω_1 and ω_2 denotes angular frequencies of waves, and k_1 and k_2 are wave vectors.

Let $\delta_1 = \omega_1 t - k_1 r_1 + \varphi_1$ and $\delta_2 = \omega_2 t - k_2 r_2 + \varphi_2$

Then $\mathbf{e_1} = E_1 \sin \delta_1$ and $\mathbf{e_2} = E_2 \sin \delta_2$

Now according to principle of superposition

$\mathbf{e} = \mathbf{e_1} + \mathbf{e_2}$

magnitude of resultant vector e

$$e^2 = e_1^2 + e_2^2 + 2\vec{e_1}\vec{e_2}$$

If at a instant of time E_1 and E_2 amplitude of waves then, resultant amplitude E is

$$E^2 = E_1^2 + E_2^2 + 2E_1E_2cos(\delta_1 - \delta_2)$$

The average intensity of light is proportional to square of amplitude

$I \propto E^2$ thus equation becomes

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle cos(\delta_1 - \delta_2)\rangle$$

In above equation I_1 and I_2 are the average intensities due to each wave. They are independent of time.

The last term in above equation is known as *interference term* which depends on time

Now

$$\langle cos(\delta_1 - \delta_2)\rangle = \frac{1}{T}\int_{t=0}^{t=T} cos\ (\delta_1 - \delta_2)dt$$

Here t is period of electric field oscillation. On substituting value of δ_2 and δ_1 in above equation

$\langle cos(\delta_1 - \delta_2)\rangle = \frac{1}{T}\int_{t=0}^{t=T} cos\ \{(\omega_1 t - \omega_2 t) + (k_2 r_2 - k_1 r_1) + (\varphi_2 - \varphi_1)\}dt$ --(1)

Case I: Incoherent sources

If angular frequency of both source is not same thus $\cos(\delta_1 - \delta_2)$ is time dependent and average value is zero. Thus superposed two waves produce the average intensity $I_1 + I_2$ at point P

Case II: Coherent sources:

For sources to be coherent there angular frequency should be same thus $\omega_1 = \omega_2 = \omega$ (say) Also since both waves are travelling in same medium there speed will be also same thus wave length is same thus $k_1 = k_2 = k$ (say) for sake of simplicity we will consider $\varphi_2 = \varphi_1$.

From equation (1) ignoring negative sign of cos

$$\langle cos(\delta_1 - \delta_2)\rangle = \frac{1}{T}\int_0^T cos\{k(r_2 - r_1)\}dt$$

$$\langle cos(\delta_1 - \delta_2)\rangle = \frac{1}{T}cos\{k(r_2 - r_1)\}\int_0^T dt$$

$\langle cos(\delta_1 - \delta_2)\rangle = cos\{k(r_2 - r_1)\}$-- eq(2)

Further we will assume that amplitude of both waves is equal $I_1 = I_2 = I'$ then

From equation (1) and eq(2) we get

$$I = I' + I' + 2\sqrt{I'I'}cosk(r_2 - r_1)$$

$$I = 2I' + 2I'cosk(r_2 - r_1)$$

$$I = 2I'\{1 + cosk(r_2 - r_1)\}$$

$$I = 2I'\left[2cos^2\left\{\frac{k(r_2 - r_1)}{2}\right\}\right]$$

[Use trigonometric identity $cos^2u = \frac{1+cos2u}{2}$]

$$I = 4I'cos^2\left\{\frac{k(r_2 - r_1)}{2}\right\}$$

Here $r_2 - r_1 = \delta$ is known as the path difference between superposing waves

$$I = 4I'cos^2\left\{\frac{k\delta}{2}\right\}$$

Special Cases

Case I : Constructive Interference

For $I = 4I' = I_0$ maximum intensity of light

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle cos(\delta_1 - \delta_2)\rangle$$

For destructive interference $\cos(\delta_1 - \delta_2)$ = -1 thus

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

Or

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

$$I_{min} \propto (A_1 - A_2)^2$$

Condition for sustained interference

The interference pattern in which the positions of maximum and minimum intensity of light remain fixed with time, is called sustained or permanent interference pattern. The conditions for the formation of sustained interference may be stated as :

(i) The two sources should be coherent

(ii) Two sources should be very narrow

(iii) The sources should lie very close to each other to form distinct and broad fringes

Solved numerical

Q) Two sources of intensity I and 3I are used in an interference experiment. Find the intensity at a point where the waves from the two sources superimpose with a phase difference (1) Zero (2) π/2

Solution:

In case of interference

$$I' = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle cos(\delta_1 - \delta_2)\rangle$$
$$I' = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle cos(\delta)\rangle$$

(1) As δ = 0, cosδ =1

$$\therefore I' = 3I + I + 2\sqrt{(3I)(I)} \times 1$$
$$I' = 4I + 2\sqrt{3}I$$

(2) As δ = π/2, cosδ = 0

$$\therefore I' = 3I + I + 2\sqrt{(3I)(I)} \times 0$$
$$\therefore I' = 4I$$

Q) Ratio of the intensities of rays emitted from two different coherent sources is α. For the interference pattern formed by them, prove that

$$\frac{I_{max} + I_{min}}{I_{max} - I_{min}} = \frac{1+\alpha}{2\sqrt{\alpha}}$$

I_{max} : Maximum of intensity in the interference fringe

I_{min} : Minimum of intensity in the interference fringe

Solution:

Given $I_1 = \alpha I_2$
Since $I \propto A^2$
Thus $A_1 = \sqrt{\alpha} A_2$
Now

$$I_{max} \propto (A_1 + A_2)^2$$
$$I_{max} \propto \left(\sqrt{\alpha}A_2 + A_2\right)^2$$
$$I_{max} \propto A_2^2\left(\sqrt{\alpha} + 1\right)^2$$

And

$$I_{min} \propto (A_1 - A_2)^2$$
$$I_{min} \propto A_2^2\left(\sqrt{\alpha} - 1\right)^2$$
$$\frac{I_{max} + I_{min}}{I_{max} - I_{min}} = \frac{A_2^2\left(\sqrt{\alpha} + 1\right)^2 + A_2^2\left(\sqrt{\alpha} - 1\right)^2}{A_2^2\left(\sqrt{\alpha} + 1\right)^2 - A_2^2\left(\sqrt{\alpha} - 1\right)^2}$$
$$\frac{I_{max} + I_{min}}{I_{max} - I_{min}} = \frac{\left(\sqrt{\alpha} + 1\right)^2 + \left(\sqrt{\alpha} - 1\right)^2}{\left(\sqrt{\alpha} + 1\right)^2 - \left(\sqrt{\alpha} - 1\right)^2}$$
$$\frac{I_{max} + I_{min}}{I_{max} - I_{min}} = \frac{\alpha + 1 + 2\sqrt{\alpha} + \alpha + 1 - 2\sqrt{\alpha}}{\alpha + 1 + 2\sqrt{\alpha} - \alpha - 1 + 2\sqrt{\alpha}}$$
$$\frac{I_{max} + I_{min}}{I_{max} - I_{min}} = \frac{2(\alpha + 1)}{4\sqrt{\alpha}} = \frac{\alpha + 1}{2\sqrt{\alpha}}$$