

LPP for Maximization of Profit in Cloth Manufacturing analyzing Graphical and Simplex method

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Abstract: The intention of any manufacturing enterprise to maximize the earnings subject to the availability of raw material, quantity of machinery and labors, man-hours, transportation of cost, processing time product on machines etc... In this paper we confirmed that how Linear programming trouble technique is used to maximization of income with respect to the given restricted resources. The Cloth enterprise is working in the place of manufacturing two distinct kinds of cloths and it's molding as per the demand. In this find out about we consider this problem examining between two ways. In these two approaches had identical optimization obtained.

Key words: Linear programming problem [LPP], BFS, Slack Variable, Surplus Variable.

Introduction

A LPP is a approach to optimize ie. Maximize or minimize the objective function. The objective feature either profit or cost function. It is a mathematical approach for discovering optimal solution when there are limited assets like machine, labor, raw material, man-hours, and different facilities. The approach of LPP is relevant to problem in which the complete effectiveness can be expressed as a linear function. The boundaries on assets expressed in term of linear inequalities or equalities. Many sensible problems in operations problems in operations research can be expressed as linear programming problems. Certain special cases of linear programming such as network go with the flow troubles and multi commodity glide issues are regarded necessary enough to have generated tons lookup on specialized algorithm for their solution. Historically, ideas from linear programming have stimulated many of the central standards of optimization concept such as Duality, Decomposition and the significance of convexity and its generalizations. Standard structure is the ordinary and most intuitive shape of describing a LPP.

Basic Definitions:

Decision Variables: The unknown variables which are used to represent products, services, projects etc. These variables are usually inter-related in terms of utilization of resources and need simultaneous solution.

Objective Function: Objective function is a linear function in terms of decision variables. The aim of the study to optimize it i.e. either maximization or minimization.

Constraints: The limitation on resources like production capacity, raw-material, labor, man-hours, machines. These resources being expressed as linear inequalities in terms of decision variables known as constraints.

$$\text{Max } Z \text{ or Min}(Z) = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

Subject to the condition

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq = \geq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq = \geq b_2$$

.....

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq = \geq b_m$$

$$x_1 \geq 0, x_2 \geq 0, \dots, x_n \geq 0$$

The linear expression Z to be maximization for profit function and to be minimization for cost function is called the objective function. The m inequalities $\leq = \geq$ are the constraints which the objective function is to be optimized.

Slack Variable: A non-negative variable which is added to the left hand side of less or equal constraint to convert constraint into equations is called as slack variable.

Surplus Variable: A non-negative variable which is subtracted from left hand side of greater or equal constraint to convert constraint into equations is called as surplus variable.

Degenerate: A basic solution to the system is called degenerate if one or more of the basic variable vanish.

BFS: A feasible solution to an LPP which is also a basic solution to the problem is called a basic feasible solution[BFS] to the LPP.

Properties:

- [1] If an LPP has a feasible solution then it is also has a basic feasible solution.
- [2] A basic feasible solution to an LPP must correspond to an extreme point of the set of all feasible solutions and conversely.
- [3] If the feasible region of an LPP is a convex polyhedron then there exists an optimal solution to the LPP and at least one basic feasible solution must be optimal.
- [4] Let an LPP have a basic feasible solution. If we drop one of the basis vectors and introduce a non-basis vector in the basis set, then the new solution obtained is also a basic feasible solution.
- [5] Any convex combination of k different optimum solutions to an LPP is again an optimum solution to the problem.
- [6] Let there exist a basic feasible solution to a given LPP. If for at least one j, for which

$y_{jj} \leq 0$ ($i = 1, 2, 3, \dots, m$) and $z_j - c_j$ is negative, then there does not exist any optimum solution to this LPP.

Simplex Algorithm

This approach or algorithm is used to attain an most appropriate solution by way of applying following step one via one to linear programming problem.

1. Convert the LPP into standard form.
2. Check whether all b_i are non-negative. If negative then multiply an equation by -1.
3. Convert all inequalities to equation using slack/ surplus variables.
4. Obtain an initial basic feasible solution $X_B = B^{-1}b$ and put it in the 1st column of the simplex method table.
5. Compute the net evaluation $z_j - c_j$ ($j=1, 2, 3, \dots, n$) using the relation $z_j - c_j = C_B Y_j - C_j$
Examine the sign $z_j - c_j$
 - a) If all the BFS of X_B is an optimum basic feasible.
 - b) If atleast one proceed on to next step.
6. If there are more than one negative $z_j - c_j$ choose the most negative.
Let it be $z_r - c_r$, $j=r$.
 - a) If all then there is an unbounded solution to the given problem.
 - b) If at least one then the corresponding vector y_r enters the basis y_B .
7. Compute the ratios $\left\{ \frac{x_{Bi}}{y_{ir}}, y_{ir} > 0, i = 1, 2, \dots, m \right\}$
and choose the minimum of them. Let the minimum of the ratios be $x_{BK} y_{kr}$
Then the vector y_k will level the basis y_B . The common element y_{kr} , which k th row and the r th column is known as the pivot element of the table.
8. Convert the pivot element to unity by dividing its row by the pivot element itself and all other elements in its column to zeroes by making use of the relations:
For $i=1, 2, \dots, m$: $j=1, 2, \dots, n$ and $i \neq k$

$$\widehat{y}_{ij} = y_{ij} - \frac{y_{kj}}{y_{kr}} y_{ir}$$

$$\widehat{y}_{kj} = \frac{y_{kj}}{y_{kr}}$$
9. Go to step 5 and repeat the computational procedure until either an optimum solution is obtained or there is an indication of an unbounded solution.

Main Result:

A company makes two different kinds of cloths. If A is high quality cloth and B is lower quality of cloth. The respective profits are Rs. 100 and Rs. 50 per cloth. Each cloth of type A requires twice as much time as a cloth of type B, and if all cloths were of type B, the company make 1500 per day. The supply of cotton is sufficient for only 1000 cloths per day (Both A & B) cloth. A requires a fancy item 500 per day are available. There are only 800 per day available for cloth B. Determine the optimal product max.

Graphical Method Solution:

Find the maximum value of the function

$$F = 100 x_1 + 50 x_2$$

subject to the constraints:

$$\begin{cases} 2 x_1 + x_2 \leq 1500 \\ x_1 + x_2 \leq 1000 \end{cases}$$

$$x_1 \geq 0 \quad x_2 \geq 0$$

Points whose coordinates satisfy all the inequalities of the constraint system are called **a region of feasible solutions**.

Step 1

Let's solve 1 inequality of the system of constraints. $2 x_1 + x_2 \leq 1500$

To plot a straight line: $2 x_1 + x_2 = 1500$

$$\text{Let } x_1 = 0 \Rightarrow x_2 = 1500$$

$$\text{Let } x_2 = 0 \Rightarrow 2 x_1 = 1500 \Rightarrow x_1 = 750$$

Plots (0, 1500) and (750, 0)

Step 2

Let's solve 2 inequality of the system of constraints. $x_1 + x_2 \leq 1000$

To plot a straight line: $x_1 + x_2 = 1000$

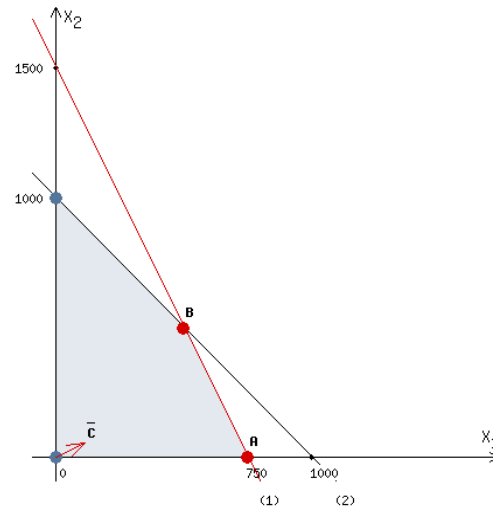
$$\text{Let } x_1 = 0 \Rightarrow x_2 = 1000$$

$$\text{Let } x_2 = 0 \Rightarrow x_1 = 1000$$

Plots (0, 1000) and (1000, 0)

Step 3

We need to plot the vector $C = (100, 50)$, whose coordinates are the coefficients of the function F. Vector C is not shown at scale because it is too short.



Step 4

We will move a "red" straight line perpendicular to vector $\vec{c}$ from the lower left corner to the upper right corner.

The function F has a minimum value at the point where the "red" straight line crosses the region of feasible solutions for the first time. The function F has a maximum value at the point where the "red" straight line crosses the region of feasible solutions for the last time. There is an assumption that the function F has a maximum value at points A and B at the same time. The coordinates of point A (750,0) are known. Let's calculate the value of the function F at point A (750,0).

$F(A) = 100 * 750 + 50 * 0 = 75000$. Let's find the coordinates of point B .

Point B is on the straight line (1) and on the straight line (2) at the same time.

$$\begin{cases} 2x_1 + x_2 = 1500 \\ x_1 + x_2 = 1000 \end{cases} \Rightarrow \begin{cases} x_1 = 500 \\ x_2 = 500 \end{cases}$$

Let's calculate the value of the function F at point B (500,500).

$$F(B) = 100 * 500 + 50 * 500 = 75000$$

$F(A) = F(B)$. Then we can conclude that the function F has a maximum value at any point on the line segment AB

Result:

$$x_1 = 750 * t + 500 * (1 - t)$$

$$x_2 = 0 * t + 500 * (1 - t)$$

$$0 \leq t \leq 1$$

$$F_{\max} = 75000$$

By Simplex Method Solution:

By introducing slack variables $s_1 \geq 0$ and $s_2 \geq 0$ respectively, the set of constraints of the given LPP are converted into the system of equations:

$$\begin{pmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 1500 \\ 1000 \end{bmatrix}$$

Using now simplex method, the iterative simplex table is

Initial iteration

		c_j	100	50	0	0
c_B	y_B	x_B	x_1	x_2	s_1	s_2
0	s_1	1500	2	1	1	0
0	s_2	1000	1	1	0	1
Z=0		z_j	0	0	0	0
		$z_j - c_j$	-100	-50	0	0

From the table, it is apparent that are two $z_j - c_j$ which are negative. We choose the most negative of these, -100. The corresponding column vector x_1 enters the basis and s_1 is leaving the basis.

		c_j	100	50	0	0
c_B	y_B	x_B	x_1	x_2	s_1	s_2
100	x_1	750	1	1/2	1/2	0
0	s_2	250	0	1/2	-1/2	1
Z=75000		z_j	100	50	50	0
		$z_j - c_j$	0	0	50	0

The above simplex table yields a new basic feasible solution with increased value of z . Moreover, no further improvement in the value of z is possible, since $z_j - c_j \geq 0$.

The maximal basic feasible solution to the given LPP therefore

$$x_1 = 750$$

$$x_2 = 0$$

$$\text{Max } Z = 7500$$

Conclusion:

In this manuscript using two different ways in the LPP of cloth manufactures of the product. It observe that using the graphical method and simplex method is same solution found. So this two method are clearly giving a same solution.

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